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Real Options Valuation of Integrative Information Systems

Thesis

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REAL OPTIONS VALUATION OF INTEGRATIVE INFORMATION SYSTEMS

Abstract

Spending on investments in integrative information systems (IIS) has considerably risen during the last few years due to a high need for linking various information systems. The demand for integrating the systems stems from developments like mergers and acquisitions and is typically satisfied in practice using Enterprise Application Integration solutions, Enterprise Resource Planning systems, Portals, or Data Warehouses. For the valuation of such an investment previous literature recommends the use of a real options analysis (ROA) since traditional capital budgeting methods such as the Net Present Value underestimate its value. Contrary, the ROA is able to conveniently account for managerial flexibility, represented by the possibility to implement follow-on opportunities, generated by the IIS. However, in practice ROA suffers from a lack of appliance mainly because of its complexity. This thesis precisely closes this gap and develops a simplified process model for a ROA by exactly tailoring the broad real options concept to the requirements of an investment valuation of IIS. For that, it reviews option pricing models from the financial world as well as previous research in the area of ROA and creates the desired model by conducting a ROA for four case studies in detail. The study reveals new findings concerning the question of how a decision-maker can apply the real options method and at the same time, when he/she is able to abandon a detailed ROA or a ROA at all.

Keywords: IT Investment Valuation; Real Options; Option Pricing Models; System Integration

Zusammenfassung

Die Ausgaben für integrative Informationssysteme (IIS) sind in den letzten Jahren beträchtlich gestiegen. Die Forderung Informationssysteme zu vereinen hat ihren Ursprung in Entwicklungen, wie beispielsweise Unternehmenszusammenschlüssen, und wird in der Praxis durch Lösungen, wie Enterprise Application Integration, Enterprise Resource Planning, Portals oder Data Warehouses, gestillt. Die bestehende Literatur empfiehlt für die Beurteilung von solchen Investitionen eine Realoptionsanalyse (ROA), da traditionelle Methoden der Investitionsrechnung, wie der Net Present Value, zu einer Unterbewertung führen. Im Gegensatz dazu ermöglicht eine ROA die korrekte Betrachtung von Handlungsalternativen des Managements, dargestellt durch die Möglichkeit Folge-Investitionen zu tätigen, die durch das IIS ermöglicht werden. In der Praxis leidet das Konzept der ROA jedoch unter einem sehr niedrigen Anwendungsgrad, der vor allem durch dessen Komplexität verursacht wird. Die vorliegende Arbeit schließt genau diese Lücke, indem sie die breit definierte Realoptions-Methode auf eine Anwendung für Investitionen in IIS zurechtschneidert und so ein einfaches Vorgehensmodell entwickelt. Dafür wird die existierende Literatur betreffend Techniken zur Bewertung von Finanzoptionen sowie hinsichtlich ROA betrachtet und das gewünschte Vorgehensmodell mittels Anwendung in vier Fallstudien entwickelt. Die Forschungsarbeit liefert neue Erkenntnisse bezüglich der Fragen, wie eine ROA durchgeführt werden kann und wann auf eine detaillierte ROA bzw. insgesamt auf eine ROA verzichtet werden kann.

Schlüsselwörter: IT Investitionsbewertung; Realoptionen; Optionsbewertungsmodelle; Systemintegration

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LIST OF ABBREVIATIONS

B2B................ Business to Business
B2C................ Business to Consumer
CAPM........... Capital Asset Pricing Model
CRM .... Customer Relationship Management
DCF .... Discounted Cash Flow
DTA............... Decision Tree Analysis
EAI .... Enterprise Application Integration
ERP................ Enterprise Resource Planning
FTE................ Full Time Equivalent
HTML........... Hyper Text Mark-up Language
ICT .... Information and Communication Technology
IIS .... Integrative Information System
IRBA .... Internal Ratings Based Approach
IS .... Information System
IT .... Information Technology
i.i.d................ Independent and Identically Distributed
MAD............. Market Asset Disclaimer
NPV .... Net Present Value
PDA .... Personal Digital Assistant
PDE .... Partial Differential Equation
R&D .... Research and Development
ROA .... Real Options Analysis
SCM .... Supply Chain Management
SQL .... Structured Query Language

MATHEMATICAL NOTATION

$B(n, p)$ .... Binomially distributed random variable defined as a sum of n Bernoulli random variables being equal to 1 with probability p

$B(a, b, \rho)$ .... Bivariate cumulative normal distribution function with upper integral limits a and b and correlation coefficient ρ

$C(S, T)$ .... Price of a call option on the underlying S with maturity T

$Cov(X, Y)$ .... Covariance of the random variables X and Y

$E[X]$ .... Expected value of the random variable X

$F(x)$..................... Cumulative distribution function of the random variable X

$\phi(x)$..................... Cumulative distribution function of the standard normal random variable X

$f(k; n, p)$............. Probability mass function of the binomial random variable X

$\varphi_{\mu,\sigma^2}(x)$ .... Probability density function of the normal random variable X

$\varphi(x)$.................... Probability density function of the standard normal random variable X

K......................... Strike price of an option

I_t .... Increment in a random walk from time t-1 to time t

$\ln(x)$.................... Natural logarithm of x

$N(\mu, \sigma^2)$ .... Normally distributed random variable X with expected value μ and variance σ^2

$N(x)$.................... Cumulative distribution function of the standard normal random variable X

μ .... Expected value of the random variable X

r .... Rate of interest

r_f......................... Risk-free rate of interest

$P(A)$.................... Probability of the occurrence of event A

$P\{X = x_j\}$............ Probability that the random variable X has the value x_j

$p(x)$..................... Probability density function of the random variable X

p_i......................... Probability that i is the outcome of an experiment

q .... Rate of dividend paid by a share

S .... Price of the underlying asset of an option

σ^2........................ Variance of the random variable X

T .... Maturity date

τ.......................... Time to maturity

U......................... Universal sample space of an experiment

$Var(X)$ .... Variance of the random variable X

W_t....................... Standard Wiener process

$\{X_n; n \geq 0\}$ .... Random walk with start at $n = 0$

Part 1: INTRODUCTION

1.1 State of the Field and Problem Definition

Many researchers have recently turned to the valuation of investments in information technology (IT). This is mainly the result of a still increasing Information and Communications Market (ICT) with annual growth rates in ICT spending of more than 5.6% a year between 2000 – 2005. Whereas the expenses in OECD countries went up 4.2% they are rising most rapidly in certain non-OECD economies. Examples therefore are China, Russia, and India with rates between 20% and 25% per year [OECD06]. Within the field of IT, investment valuation is especially important for integration projects. For years, investments in IT infrastructure have been acquiring a major part of IT's expenditures. Weill and Broadbent [WeBr98] conducted a study among a sample of large companies and found that 40% of the IT capital outlays were dedicated to infrastructure. A more current survey among 1,270 organisations shows that in 2005 they planned to **spend most of their money on specific IT infrastructure investments, namely application integration** [BaMa05]:

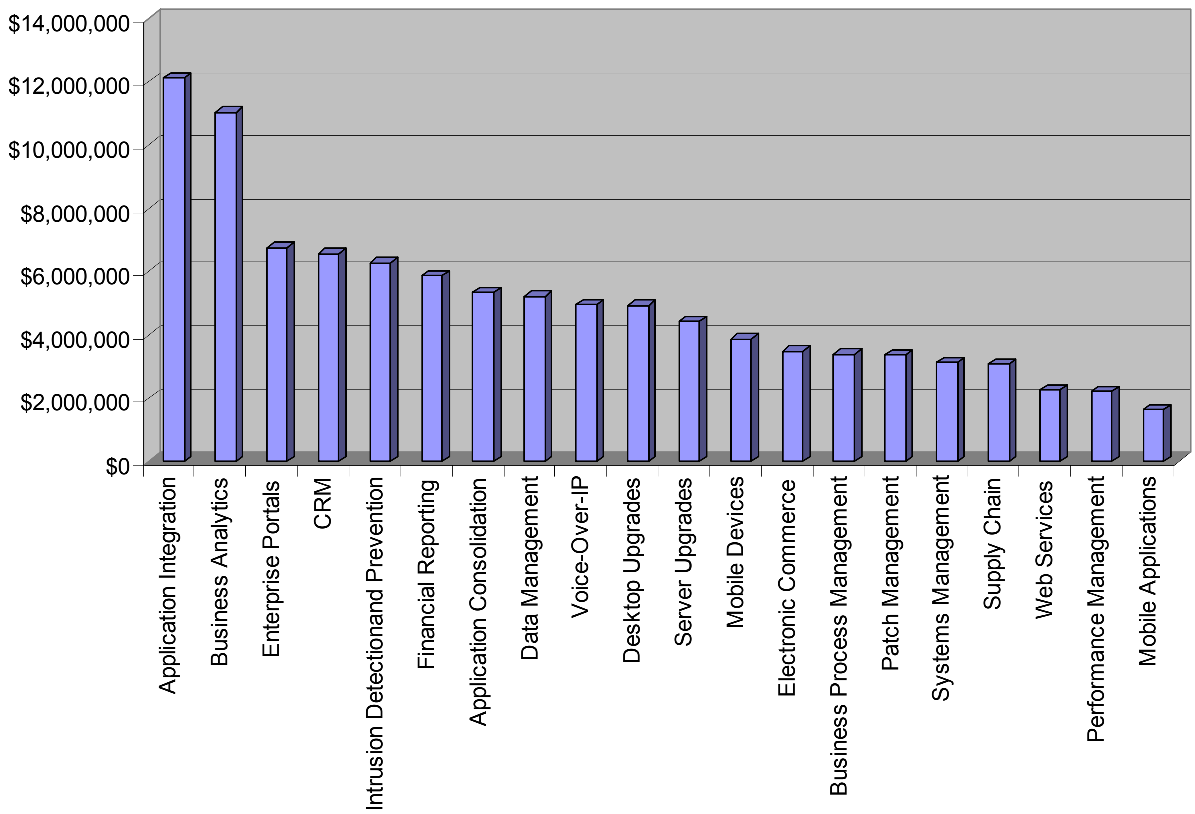


Figure 1: Planned spending on IT projects in 2005
Source: Own representation based on [BaMa05]

Hence, managers are looking for improved ways not only to measure the benefits of IT investments but also to understand them [DaKM07]. In general, companies analyse their investment decisions using an internal rate of return or a net present value analysis [GrHa01]. Previous research has already demonstrated that these methods undervalue infrastructure investments since they do not account for follow-on opportunities suitably. This raises big problems due to the fact that the benefits of an IT infrastructure mainly arise from such possible follow-on projects.

As an alternative instrument for the valuation of investments in IT infrastructure various authors recommend a real options analysis (ROA) (e.g. [DoBr91], [TaFM00], [TrLe00], [DaKM00], and [DaKM07]). However, the real options approach is still suffering from a lack of application in practice. Using the results of a survey of Peemöller et al. [PeBK02], Hilpisch [HiYv06] points out that this is not necessarily a result of a missing publicity:

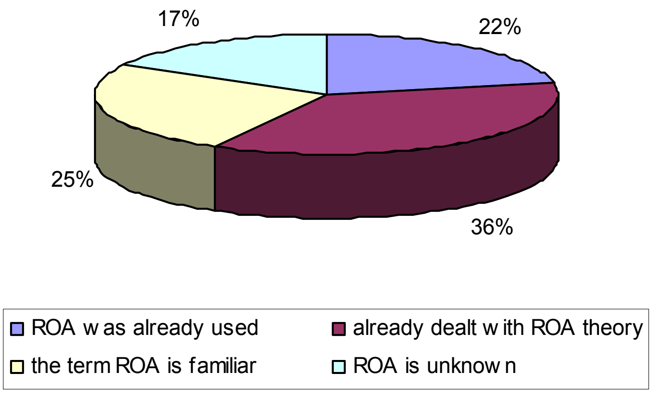


Figure 2: Familiarity of ROA
Source: Own representation based on [HiYv06]

They investigated 36 German companies and found out that ROA is completely unknown to only 17% of them. According to the results of the study **the major problem is the high complexity of a ROA** followed by missing experience and a low spread of use:

#	Barrier	True
1	High complexity of ROA	52%
2	Missing practical experience	43%
3	Low spread of use	43%
4	Other techniques serve the same purpose	30%
5	Difficult estimation of the parameters necessary	26%
6	Difficult identification of the correct option	21%
7	High effort when applying	17%
8	Missing background in financial options theory	17%

Table 1: Barriers for conducting a ROA
Source: [HiYv06]

Tallon et al. [TKLWZ02] support this view with reference to a survey published in the Economist [Econ00]. Here, 50% of American firms which experimented with ROA rejected it because of its complexity and underlying assumptions. Nevertheless, they trust in the advantages of a ROA and point out that in the mid 1960's even the NPV method was rejected for having unrealistic assumptions and for being overly complex.

All in all, the dilemma is as follows: on the one hand, there is the need to value investments in integrative information systems (IIS) because of their importance in a growing market. On the other hand, there exists a method, namely ROA, which overcomes major shortcomings of traditional budgeting techniques. Although ROA perfectly fits a consideration of follow-on opportunities generated by an IIS, its actual degree of application is very low.

1.2 Research Question and Objectives

A big problem when applying a ROA to an investment in an IIS stems from the fact that the real options concept is designed for various industries and a wide range of different option types. Consequently, there exist many models but it remains unclear when to use which one. The purpose of this thesis is to develop a simple process model for a ROA by concentrating on a specific application, namely on the investment valuation of an IIS. This leads to the following research question:

> *How can a ROA be simplified for the valuation of an investment in an IIS to enable a comfortable implementation in practice?*

In order to answer the research question it is necessary to consider four research objectives:

> *1. Review of the fundamentals of the real options technique, i.e. financial option pricing, and previous research in the field of ROA*

When conducting a ROA most researchers use techniques that were originally developed for the valuation of financial options. Hence, the thesis describes these techniques and their various valuation models in detail. Moreover, it presents a summary of the outcomes of previous research in the field of ROA.

2. Development of a comprehensive process model for the ROA of investments in IIS

Second, it has to be explained how the various valuation models can be applied for a ROA of investments in IIS. Important issues in this context are to consider the follow-on opportunities of the IIS and to estimate the input parameters. For this a comprehensive process model is developed which describes the course of action and contains complex as well as simple valuation models.

3. Application of the comprehensive process model to case studies

It follows the deployment of the comprehensive process model on case studies in order to demonstrate a ROA in practice and to search for possible areas of simplification.

4. Analysis of the case study findings and creation of a simplified process model

The fourth objective concerns the analysis and discussion of the case study findings. The aim is to create a simplified process model out of the comprehensive one.

To summarise, the thesis addresses the problems from above by providing a process model for the valuation of IT integration investments which uses a tailored ROA to suitably account for the generated follow-on opportunities.

1.3 Methodology and Structure of the Thesis

The research question from above is of a "how"-character and for its investigation this thesis chooses an exploratory case study approach with multiple cases. Bennet [BeAn01, p. 1513] defines a case study as follows:

"A case study is thus a well-defined aspect of a happening that the investigator selects for analysis, rather than a happening itself."

This type of research, where the focus is set on interconnections among parts and aspects within single cases differs from the variable-oriented approach. There, the goal is to find cross-case patterns without first understanding each case itself. However, choosing the case study approach does not mean that the researcher only carries out a within case-analysis. He also uses cross-case analyses to investigate e.g. commonalities between the cases. Here exists again a fundamental difference between the case study approach and the variable-oriented approach as the latter one mainly deals with questions of "why". For that, the aim is to find those variables which explain the outcome best. Otherwise, the former one addresses the question of "how" or "how does it happen". So, unlike the correlational research, the causal conditions do not compete with each other (Which one is the best?) but they combine or fit together [RaCh01]. According to Yin [YiRo03], case study research can also answer questions of "why" and not only of "how". It is important that the generalisation gained from the case studies is not confused with the as he calls "statistical generalisation". There, it is possible to make an inference about a population on the basis of empirical data collected from a sample. As cases are no sampling units, Yin recommends an "analytic generalization": the researcher uses a developed theory as a template to which the case studies' results are compared. However, regarding the current research question, the main question is how a decision-maker can use a ROA in order to value an IIS investment.

Generally, the underlying epistemology of a qualitative field research, like the case study approach, can be positivist, interpretive, or critical. Yin chooses a positivist view [MyMi97] and so does the current thesis. Basically, he discriminates three different purposes of a case study and six possible structures:

Type of structure	**Purpose of the case study**		
	Explanatory	**Descriptive**	**Exploratory**
Linear-analytic	X	X	X
Comparative	X	X	X
Chronological	X	X	X
Theory-building	X		X
"Suspense"	X		
Unsequenced		X	

Table 2: Possible purposes and structures of case studies
Source: [YiRo03, p. 152]

Exploratory case studies are used if the research field is new and unknown. Another purpose of case study research can be the description of a specific phenomenon under investigation. Moreover, the third type goes a step further than the descriptive one and already tries to explain the phenomena. All in all, it is possible to assign the current research to the class of exploratory case study research. Regarding the different structures, the thesis uses a "linear-analytic” one. Yin defines the sequence of the involved subtopics of this standard approach as follows [YiRo03]:

- the issue or problem of the study
- a review of the relevant literature
- the methods used
- the findings from the data collected and analysed
- the conclusions and implications from the findings

Some authors clearly attribute case study research to qualitative research [BeAn01], [MyMi97] what consequently leads to the use of qualitative data collection methods. Nevertheless, Yin [YiRo03] stresses out that case study research is not limited to these methods. According to him, also quantitative evidence serves as a possible data source. Normally, case study researches use interviews and documentary materials as source for the collection of relevant data [MyMi97]. Additionally, there exist also other sources, like archival records, direct observations, participant-observations, and physical artefacts [YiRo03]. This thesis concentrates on the common sources, more precisely on

- documentation and
- interviews,

which show the following advantages and disadvantages:

Source of evidence	**Strengths**	**Weaknesses**
Documentation	• stable can be reviewed repeatedly • unobtrusive not created as a result of the case study • exact contains exact names, references, and details of an event • broad coverage long span of time, many events, and many settings	• retrievability can be low • biased selectivity, if collection is incomplete • reporting bias reflects (unknown) bias of author • access may be deliberately blocked

Source of evidence	Strengths	Weaknesses
Interviews	• targeted focuses directly on case study topic • insightful provides perceived causal inferences	• bias due to poorly constructed questions • response bias • inaccuracies due to poor recall • reflexivity interviewee gives what interviewer wants to hear

Table 3: Strengths and weaknesses of data collection sources
Source: [YiRo03, p. 86]

When using documents it should always be clear that they were originally written for a specific target group and a specific purpose. As this group or purpose is normally different to the one of the case study, the investigation must always be well thought-out. Moreover, the most important use of documents is to complete and supplement evidence from other sources like interviews. They usually appear in three different forms: the first one, open-ended interviews, are designed for leaving much freedom to the respondent and only a few questions are prepared before the interview. Second, focused interviews are still open-ended and very conversational but concentrate on one topic being examined in detail. The third type, structured interviews, has more standardised questions without being fixed at all. Usually the interviewer uses a more or less specified interview-guideline [HoCh00], [YiRo03]. More details about the actually used collection methods are available in section 4.3.2. However, the opportunity to collect data from different sources is a major strength of the case study data collection. Every finding will be more substantiated and more robust if it is based on more than one evidence. Yin [YiRo03] calls this "data triangulation" pointing out that real triangulation is reached when more than one source addresses the same fact ("convergence") and not if different sources address different facts ("non-convergence").

Based on Yin's linear-analytic structure the organisation of the remaining thesis is as follows:

1. Definition and description of IIS and ROA
2. Previous research in the field of ROA of investments in IT systems or IIS
3. Formulation of assumptions based on existing research
4. Development of a comprehensive valuation model
5. Application of this model to the cases for investigating the assumptions
6. Simplification of this model based on the findings of the cases
7. Conclusions and implications for future research

Part 2: DEFINITIONS

2.1 Integration

Recently, there has been wide interest in the field of integration. The term arises in many combinations like information integration, application integration, process integration, information system integration, data integration, etc. But what hides behind these terms and what are the reasons for or drivers of the high appearance of the phenomenon “integration”?

First of all the term “integration” has to be defined. Very generally, it refers to the process of coordinating and unifying disparate elements into a whole [ReAr96]. In more detail, the boundaries can be set within four dimensions, namely level, scope, direction, and time:

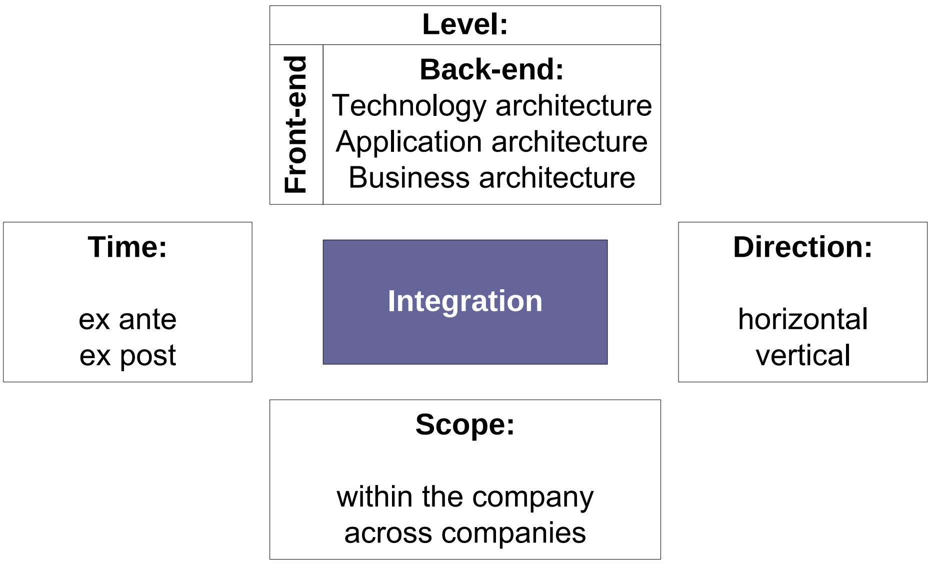


Figure 3: Definition of the term integration
Source: Adapted and extended from [KaMi02], [LeMP03], and [ScWi02]

The probably most important distinction refers to the level of integration. As a basic principle, integration can be carried out via a common front-end for the systems involved or by incorporating the various back-ends. In the first case, one common front-end controls the individual systems but they do not interact in any other way. Hence, the aim is to provide the user with the opportunity to use several systems with only one interface. In the latter case three alternatives are possible: first, technical integration gives attention to the IT which is required to achieve integration. Thus, this part focuses on the connections between the various systems and the data, i.e. bits and bytes which are transferred between them. The second section, namely application architecture concentrates on the functions of information systems which are needed to realise the business goals. Finally, the business level regards the organisational and managerial issues of the whole enterprise. So the latter point also includes business-process integration which refers to the establishment and maintenance of continuous processes across the applications [JhMP02].

Another dimension accounts for the scope of the integration's implementation and regards whether it stays within the company or conquers this boundary. In the second case the participating companies can either be at the same stage in the supply chain or at different ones. Integration across companies in a narrow sense includes operations between different companies whereas in a wider sense also processes between two subsidiaries of the same parent company are considered. Furthermore, one can distinguish between ex ante and ex post integration. Ex ante means that the implementation starts on a plain ground completely replacing the existing systems, whereas the ex post approach uses the present architecture.

After having shown the meaning of integration, the question arises wherefrom and why such a project emerges. An integration project can be started because of a strategic plan or a concrete problem e.g. of an organisational department [KaMi02]. In the first case the project is fully embedded in the company's strategy space, thus this is the preferred one. According to Ward and Griffiths [WaGr96] this strategy space consists of the following layers: business strategy (Where to go and why?), IS strategy (What is required?), and IT strategy (How can it be delivered?). The reasons for the demand for integration can be very different. The following figure gives a brief outlook on possible drivers:

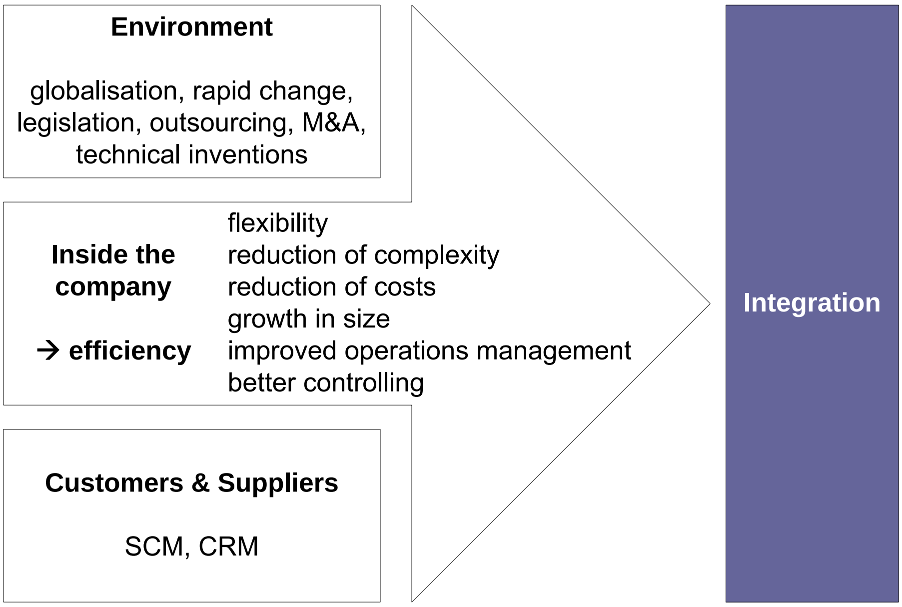


Figure 4: Reasons for the demand for integration
Source: Extended from [DLPR02]

To summarise, the company expects from integration-implementations an improvement of its own performance by better using the existing resources and/or having a better set-up for the environment, e.g. to be able to quickly react to changes. Both issues ultimately result in competitive advantages.

2.2 Integrative Information Systems

2.2.1 General Information

Taken the above definition of integration the goal of integrative information systems is the controlled connection of individual parts in order to enable communication between them. In case of an ex ante integration these parts might be located in one single system whereas in case of an ex post integration these parts represent individual systems. At a first glance it looks quite attractive to incorporate everything in one new system but this approach only works up to a specific extent. It is simply impossible to buy or develop a single system for the whole enterprise. Additionally, the individual departments will not give up their flexibility to choose their most appropriate solution. Even the great ERP systems do not cover the whole range of requirements and are subjects of integration projects. In many cases integration is realised between different companies surely leading to the involvement of various systems. Moreover, ex ante and ex post integration do not conflict with but complement each other.

Another important issue in this context are legacy-systems. The term refers to systems which were developed with a software technology other than the latest one [SnHa02]. Here the question arises whether to replace them by a new system or to integrate them. According to Sneed [SnHa02] in the past the goal was to replace them as soon as possible but nowadays integration-solutions are available which offer great services and thus reduce the need for a substitution. In his opinion replacements only make sense in severe situations like missing personnel for maintenance, discontinued support, or a non-fulfilment of the user's requirements.

In general, an integration solution has to deal with three problem dimensions as depicted in the next figure. Since it is impossible to entirely solve each of them and also not desired to eliminate one of them, the goal must be a compromise depending on the individual situation.

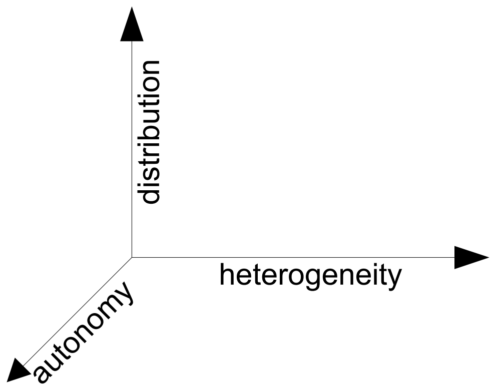


Figure 5: Problem dimensions of integration
Source: Adapted from [HaWi00]

Much of the existing distribution in the system landscape of a company is due to the existence of the already mentioned legacy systems. Techniques like "Remote Procedure Invocation" or "Messaging" help to overcome the phenomenon of distribution and are explained in the following section. Second, heterogeneity occurs at various levels and for various reasons. On a technical level, it is the result of different hardware platforms, operating systems, database management systems, and programming languages. On a conceptual level, it is the result of different programming, unequal data models, and a missing common understanding of the same real world problems. Examples for that are homonyms, i.e. the use of the same name to denote different concepts, and synonyms, i.e. the use of different names for the same concept. Encountering heterogeneity is a very difficult task which is typically tried to by common programming and data models. Third, autonomy conflicts in most cases with integration and is usually reduced by organisational changes accompanied by technical means. Systems can be autonomous in their design (programming models, naming concepts, …) as well as in their communication and function.

2.2.2 Integration Techniques

Over time experts have introduced the following main techniques to enable communication between information systems [KaMi02], [HoWo04], and [ScWi02]:

a. Front-End Integration Techniques
    1. Screen-Scraper
    2. Portal
b. Back-End Integration Techniques
    3. File Transfer
    4. Shared Database
    5. Remote Procedure Invocation
    6. Messaging

The above list shows an increasing order of sophistication but also a growing complexity. Each of the six techniques has its advantages and disadvantages and thus depends on the individual situation which fits best. In many cases even a combination may be the finest solution.

2.2.2.1 Screen-Scraper

The original purpose of a screen-scraper was to get information out of mainframe computers in order to make it accessible for client-server systems. They provide a graphical user interface which looks like the screen of a mainframe-terminal and includes an in-memory buffer.

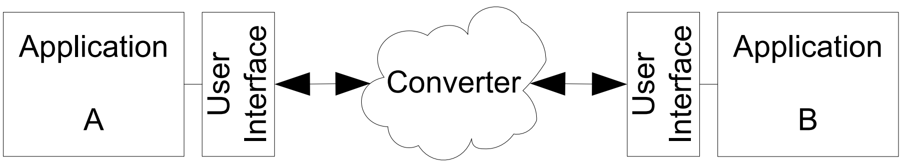


Figure 6: Screen-Scraper
Source: Own representation

When the mainframe application produces an output a special program converts this into an update of the graphical user interface. Otherwise, when the user enters information into the interface the program converts this into the appropriate edits of the character buffer [MyBr95]. New scrapers use the HTML language to extract information from one website and deposit it into another website or a database [WeKi00]. A more advanced front-end integration-technique is the following portal.

2.2.2.2 Portal

In general a portal is a web-application which provides its user with an interface to access information of various sources. The user can be an employee of a company but also a customer or another business partner and he/she has the possibility to personalise this interface [ChCh02]:

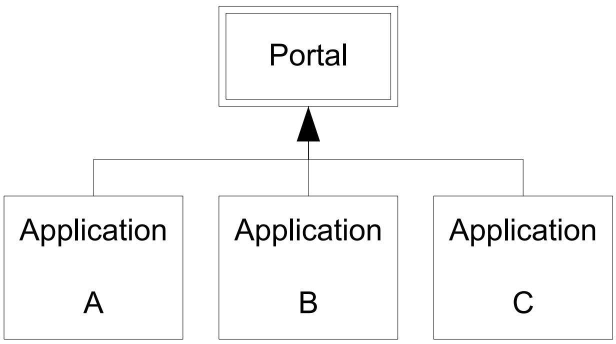


Figure 7: Portal
Source: Adapted from [HoWo04, p. 6]

Primarily, integration is realised on the front-end because only at the portal it is possible to use the applications' information. Each application can be presented in a separate frame of the web-application or they can be combined for a representation in one frame. Portals usually offer functions to communicate between different frames [WeDB02] like a drag-and-drop functionality of a marked text: the user marks for instance the article number of the frame "Articles in Stock" and moves it to the frame "Article Information" where then detailed information about this article appears.

Nevertheless this sort of front-end integration also needs communication on the back-end. The applications do not communicate with each other directly but each application has its own individual interface to the portal. Thus, at the front-end a 1:n relationship is realised whereas at the back-end only a 1:1 relationship exists. Consequently, if the portal has to provide information of many applications problems will arise. As an individual interface for each application is required the effort for the development and the maintenance of them soon becomes quite high. However, this problem can be solved as the back-end integration techniques listed in this chapter are all possible for the communication between the portal and its applications.

2.2.2.3 File Transfer

Considering this approach, communication is realised by exchanging one or more files. The file contains the information provided by application A and needed by the other application B.

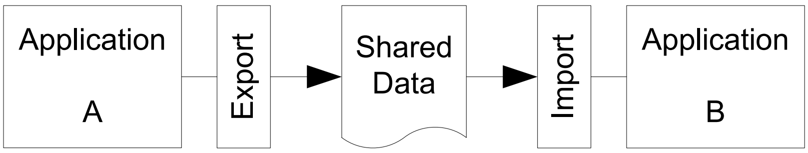


Figure 8: File Transfer
Source: [HoWo04, p. 44]

To enable integration it is necessary to transform the files into the required formats and to decide when to produce the files and when to consume them. Usually such a point in time is coupled with a specific event, such as the system's batch-run at the end of the day. The great advantage of this technique is that the integrator does not need any knowledge about the internals of the involved applications and that the transport of a very high amount of data does not lead to any problems. Unfortunately this simplicity incorporates also some disadvantages: first, the integrator has to care about technical issues like locking mechanisms (no consumption before production is completed) or the deletion of old files because there is no software package providing helpful tools. The biggest problem refers to the timeliness of the systems as infrequent updates lead to a lack of synchronisation and thus result in problems on the user-side and/or on the technical side. Someone changes e.g. the telephone number of a customer is changed in application A but not in application B. So the user of application B will be misled and when the synchronisation is done the technician has to solve the problem whether application A or B contains the correct data.

2.2.2.4 Shared Database

The usage of a Shared Database provides all participating applications with a single data source. Consequently, synchronisation and its upcoming issues are no problem any more.

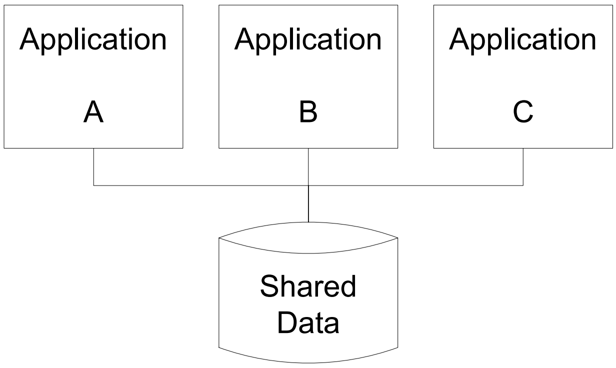


Figure 9: Shared Database
Source: [HoWo04, p. 48]

Another advantage stems from the unique data model which solves the problem of semantic dissonance, i.e. all applications have the same understanding of all fields within the database. Additionally, SQL has been accepted in practice as a common standard to get data out of the database and is supported by nearly every provider. Difficulties arise in the creation of the data model, especially when several departments are involved. Then the creation might become a political affair that will not ease the already existing technical challenges. Applications bought from external software producers bring another problem with them as they only work with their own data model where integrators have a very small space for changes. Moreover, if the database serves as source for many applications it will become a performance bottleneck caused by many read and or write operations. Deadlocks and lock-outs are the logical effects especially in case of a cross-company integration. For such situations Distributed Databases may be a solution but they entail other problems. For instance, a deadlock in a Distributed Database creates much more headache for the integrator.

2.2.2.5 Remote Procedure Invocation

Remote Procedure Invocation or Remote Procedure Call refers to a technique where one application uses the functionalities of another one by a call of an offered function:

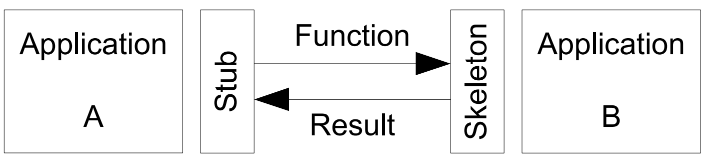


Figure 10: Remote Procedure Invocation
Source: [HoWo04, p. 50]

A real benefit is the clean implementation of encapsulation: each application has unrestricted control over its own data. If application A wants to change data of application B then this modification is not done directly but A calls B's function "change data". Thus, a change in the data structure does not affect other applications which participate in the integration solution. Another advantage offers the possibility to create different interfaces to the same data which reflects a very elegant solution to deal with semantic dissonance. Problems arise due to the fact that the applications are still coupled quite tightly. The different calls between each other can quickly grow into a knotty network. Remote procedure calls are quite convenient for developers because they work similar as local calls do. Unfortunately, they lead to much more problems in terms of performance and reliability: an invocation across the network is simply much slower and more vulnerable to errors. Moreover, too many calls can overload the system resulting in a deadlock.

2.2.2.6 Messaging

Compared to the former techniques Messaging represents the most sophisticated one. One of the basic elements is the message which consists of two parts. First, the header comprises meta-information relevant for the messaging system and second, the body contains the data. The message is produced by a sender (producer) and then transferred to the receiver (consumer) using a channel. Moreover, a messaging system controls the whole process and a message endpoint connects the participating applications to the messaging system. If they do not agree on the same data format a message translator can convert them.

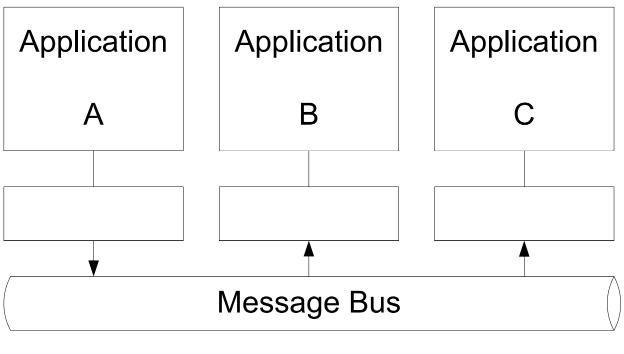


Figure 11: Messaging
Source: [HoWo04, p. 54]

In a nutshell the transmission of a message follows five steps:

1. Create

   The sender creates the message and fills it with data.

2. Send

   The message is added to a channel by the sender.
3. Deliver

   The messaging system transfers the message and makes it available for the receiver.
4. Receive

   The message is read by the receiver from the channel.
5. Process

   The receiver extracts the data from the message.

In order to enable a reliable communication messaging uses three main concepts representing a real advantage of the technique. The first concept, "send and forget", depicts the sender's autonomy since the messaging system guarantees for the delivery after sending and so the sender is free and available for new work. Thus, the possibility of asynchronous communication is offered allowing the sender and the receiver to handle the messages at their own speed. If the receiver needs more time this will not lead to problems as all messages are queued until it is ready for the processing. The second concept, "store and forward", describes the process of storing the message before forwarding it from computer A to B where it is stored again and if necessary also forwarded. This results in a very reliable communication because in case of problems the messaging system resends the message until it succeeds. Finally, the so called "loose" coupling is implemented very well, i.e. the participating parties do not make many assumptions about each other when they exchange information. Hence, changes in the application's data structure do not compromise the communication.

To summarise the advantages, this technique is somewhat similar to File Transfer since it is possible to transform the messages during the transport without any notification by the applications. Improvements are that data packets are produced quickly and that the receiver is notified automatically. Additionally, it is better encapsulated than Shared Database and more reliable than Remote Procedure Invocation.

Otherwise, the high degree of sophistication leads to a higher complexity, e.g. developers are forced to work with a complicated event-driven programming model. The situation is even more difficult if messages depend on each other and their consumption has to take place in a specific order. Without any additional effort, messaging systems only guarantee the delivery but not the point in time. An additional drawback might be the performance because this type of communication requires some overhead which slows down the whole communication. Moreover, compatibility can issue some more challenges as many systems and platforms are not ready for messaging and many vendors only support their own protocols. Hence, the situations require message bridges which integrate the various integration solutions.

2.2.3 Integration Approaches

Using the techniques from above, six main approaches concerning IIS are currently present in practice. This section provides a definition of each approach and gives in addition a brief explanation including a classification according to Figure 3. As it is not necessary for the investment valuation of this thesis to go further into detail, the interested reader is referred to technical literature for more elaborate information.

- Portals
- Enterprise Application Integration (EAI)
- Enterprise Resource Planning (ERP)
- Data Warehousing
- Individual Solutions
- Combined Solutions

In general a portal is a system which allows its user to receive any corporate information he/she needs via a web browser. As a flexible information technology platform it enables a link between different sources of information and provides the user with the possibility to customise his/her needs [ChCh02]. The integration occurs on the front-end independently from the back-end systems leading to a large variety of portals in practice. Kemper et al. [KeMC06] mention three main types of portals: first, public portals are classic web-portals which are available in the internet and provide a range of information services. Second, personal portals are also free accessible but concentrate their offering on devices like mobile phones and PDAs. The last type, corporate portals, provides internal and external parties with information about a company and thus they are especially in use in fields like B2C, B2B, or B2E. All in all, a portal enables for an ex post integration across or within a company.

Considering EAI, the main challenge is to enable existing systems to communicate with each other [LeSH03]. So here integration is done ex post and with no constraints in the two dimensions scope and direction, i.e. within or across companies. While at the beginning EAI was seen as a strategy which only addressed the technical and the application's architecture [HaWi00], in the last few years the focus of EAI evolved [GrSt03]. The result is a view including the process/business level as well as the former two [KaMi02], [LeSH03].

The central issue of the ERP concept is to reach a situation of seamless integrated processes across functional areas. Hence, it allows for an improved workflow, standardisation of various business practices, better order management, accurate accounting of inventory, and improved supply chain management. ERP systems like SAP are the vehicles which enable the accomplishment of these challenges [MaSV00]. Such a software package provides a company with best practice processes that are adapted, or as it is called in jargon "customised", to the company's specific needs [StPi98]. This approach builds a complete new organisational and technical infrastructure within the company concerned. However, beside the clear ex ante and business architectural character, the scope of ERP is changing. At the beginning the focus was set to the own company but trends like B2B, B2C, SCM, and CRM extend basic ERP systems. The latest solutions also offer tools for supply chain management across different companies [JaBe03].

A data warehouse represents a collection of data to support the management's decision-making process. The data collection is subject-oriented instead of product-oriented and origins from different operating systems. It is collected over time and non-volatile or, in other words, persistent [ZeCY03]. Thus, data warehousing belongs to the integration level of application architecture. As it uses existing systems, the implementation is executed ex post. Originally, a data warehouse only contains intra-company data but the internet has led to a major change. This technology allows connecting a much larger community of users to the data warehouse who want to analyse data from every location within a group of companies or from different partners of the company [ZeCY03].

Finally there remain two additional approaches to integration. One addresses individual solutions where mostly a point-to-point connection via the Web is established in order to connect two systems. Of course, such a solution is only designed for a particular problem and it can be a quick solution for this one, but in most cases it refers to a missing organisation of the company's system-landscape [KaMi02]. The last approach refers to combinations of the previous ones like EAI of ERP systems [LeSH03] or ERP and data warehousing [ZeCY03].

Here the goal is to join the advantages of the individual approaches by minimising the disadvantages in order to reach the best solution of the particular problem.

2.3 Investment Valuation

2.3.1 General Information

The implementation of an integrative information system represents an investment, i.e. it generates a stream of cash inflows and outflows. In order to enable managers to decide about an investment they need a correct valuation of the investment and an appropriate decision rule. In the literature, investment decision rules are usually referred to as capital budgeting techniques. According to Copeland, Weston, and Shastri [CoWS05] the main requirement on the techniques is that they maximise the shareholders' wealth. All in all they impose the following claims:

1. All cash flows should be considered
2. The cash flows should be discounted at the opportunity cost of funds
3. The technique should select from a set of mutually exclusive investments the one that maximizes shareholders' wealth
4. Managers should be able to consider one investment independently from all others

Graham and Harvey [GrHa01] conducted a survey where they examine the practice of corporate finance in the areas of capital budgeting, cost of capital, and capital structure. The sample is cross-sectional and consists of approximately 4,440 firms from the U.S. and Canada. Using the answers of 392 chief financial officers the authors are able to rank the different investment valuation techniques considering their degree of usage:

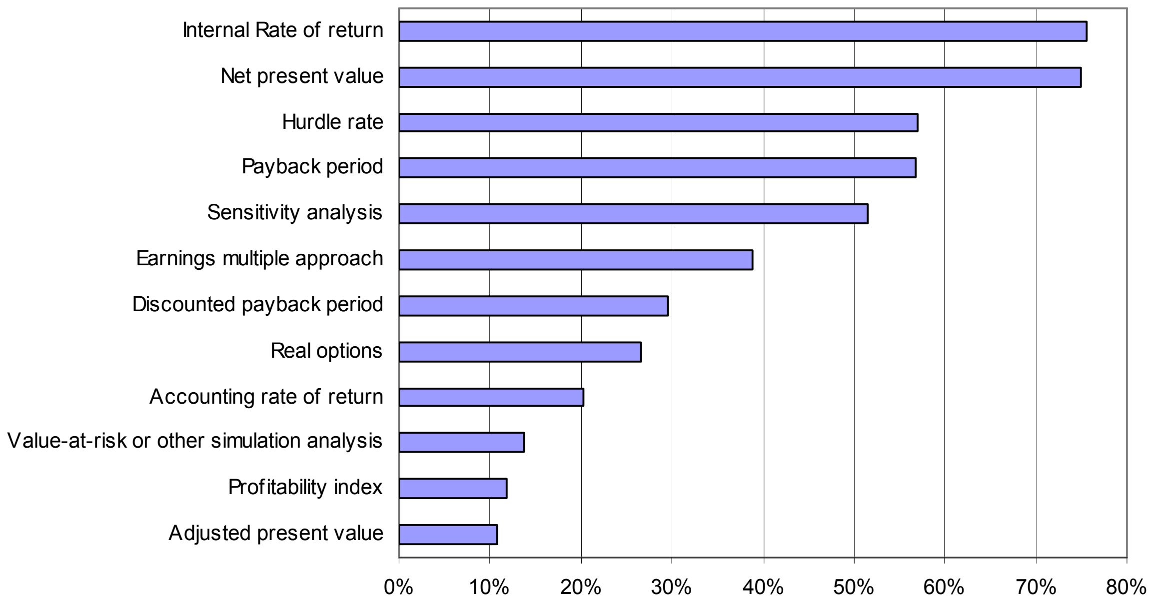


Figure 12: Percent of CFOs who (almost) always use a specific valuation technique
Source: Own representation based on [GrHa01]

The figure shows that traditional approaches like the internal rate of return or the net present value are the most common ones. Modern techniques like the real options method are not as widespread because of their increased complexity. However, investment decisions are grouped into two categories, which are explained in the following two sections:

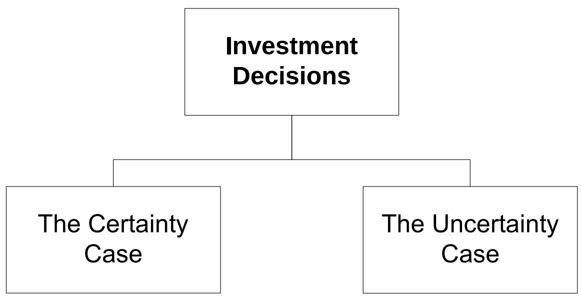


Figure 13: Categories of investment decisions
Source: Own representation

2.3.2 The Certainty Case

In the certainty case all future cash flows of the investment are known. Such a situation relieves the investment valuation but unfortunately it occurs very seldomly in practice. However, literature mentions the following techniques [CoWS05]:

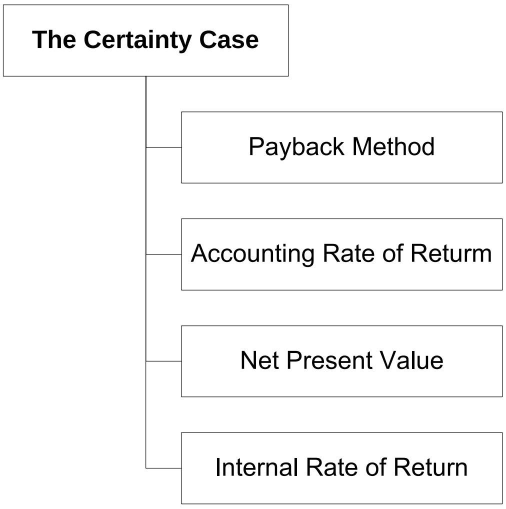


Figure 14: Capital budgeting techniques – the certainty case
Source: Own representation

As, at the time the of the decision, the cash flows of an investment in IT infrastructure are not known for sure this study goes not into detail concerning these techniques. The interested reader is referred to basic literature in finance like [BrMy03], [CoWS05], or [GHLN06].

2.3.3 The Uncertainty Case

The investment decision becomes considerably more difficult with the existence of risk, i.e. if the future cash flows are uncertain. The present research investigates the following approaches for the valuation of IT infrastructure investments under risk:

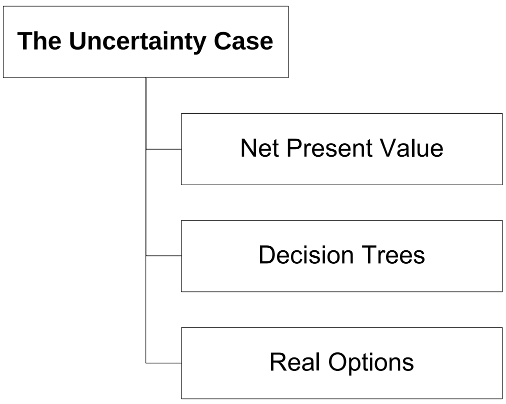


Figure 15: Capital budgeting techniques – the uncertainty case
Source: Own representation

For details on these techniques please see the following chapters: 3.1.1 Net Present Value, 3.1.2 Decision Trees, and 3.3 Valuation of Real Options.

Part 3: THE REAL OPTIONS METHOD

3.1 Traditional Methods and their Problems

As depicted above the study investigates three methods for the valuation of investments in integrative information systems. These techniques are

1. the net present value (NPV),
2. the decision tree analysis (DTA),
3. and the real options analysis (ROA).

Moreover, the financial options method is also introduced as it represents a prerequisite for the real options approach. The next sections will explain each of the techniques in detail revealing that the usage of the traditional methods (NPV and DTA) leads to many problems which however can be overcome with a real options analysis.

3.1.1 Net Present Value

3.1.1.1 Information about the Method

The NPV technique has three major features. First, it recognizes that an euro today is worth more than an euro tomorrow. Second, it depends on the forecasted cash flows from the investment and the opportunity cost of capital. Finally, all cash flows are measured in today's euros enabling them to be added up.

For a determination of the present value of the future cash flows C_t it is necessary to discount the cash flows back to present:

$$PV_0 = \sum_{t=1}^{n} \frac{C_t}{(1+r)^t}$$

This is referred to in the literature as "the discounted cash flow (DCF)" formula [BrMy03]. The NPV is received by adding up all present values of each cash flow which is produced by the investment. If the resulting NPV is greater than zero the investment decision is positive [CoWS05].

$$
\begin{aligned}
NPV &= C_0 + PV_0 \\
&= C_0 + \sum_{t=1}^{n} \frac{C_t}{(1+r)^t} \\
&= \sum_{t=0}^{n} \frac{C_t}{(1+r)^t}
\end{aligned}
$$

The parameter r of the discount factor, defined by $1/(1+r)^t$, represents the rate of return which equivalent investment alternatives in the capital market offer. The rate is also called the opportunity cost of capital because it is the return foregone by investing in the asset rather than investing in securities or other opportunities. If the investment takes place in a certain world, r equals the risk-free rate of interest r_f. In the uncertain case one has to adjust r for risk, i.e. to increase it by a risk premium. Hence, risk reduces the NPV and a save investment is more valuable than a risky one. When determining the risk premium it is important to consider that the premium compensates for the risk borne per period. Consequently, cash flows which are in the future have no need for a higher rate than earlier ones. They are already discounted at a superior factor because of the parameter t in $1/(1+r)^t$ [BrMy03].

Many companies estimate the rate of return required by investors in their securities and then use this company cost of capital to discount the cash flows on new investments. As Brealey and Myers point out, this might be an appropriate way for average investments. If the investment's risk differs from the company's risk, a need for an investment-specific risk-adjustment arises. A common way to estimate the company cost of capital is the CAPM (Capital Asset Pricing Model). This model also allows for the consideration of investment-specific risks by setting individual betas for the various investments. A very good illustration of the CAPM is available in [BrMy03] and [CoWs05].

3.1.1.2 Pros and Cons

The NPV method is rather simple to calculate and offers a clear decision rule. The easy application is clearly one of its strengths and leads to the high degree of usage in practice (see Figure 12 on page 21). The difficulties mainly arise when trying to assess a proper discount rate for the individual investments.

In the context of investments in integrative information systems, two major disadvantages occur:

1. NPV does not account for managerial flexibility
2. High uncertainty leads to a low NPV

The value of the integrative software-platform stems mostly from the applications which are, or will be, implemented on it. Hence, the negative NPV of the platform investment might turn positive with a consideration of the follow-on opportunities. Unfortunately, a simple sum of all NPVs is not a correct result because it is not an obligation but only a possibility to undertake the follow-on opportunities. The NPV method does not account for these possibilities for the management, often referred to as managerial flexibility or options. It assumes pre-commitment and allows for no flexibility in decision making at all. Moreover, the high degree of uncertainty about the follow-on opportunities increases the discount factor and thus decreases the NPV. As the real options approach will show uncertainty is not a burden but offers possibilities which might enlarge the investment's value.

All in all, traditional capital budgeting methods like the NPV underestimate the investment's value. In other words, the traditional NPV is only a lower bound of the real value of the investment [AmKu99], [BrMy03], [CoWs05], [ScMa03], [TaAl98], and [TrLe05]. However, according to Amram and Kulatilaka [AmKu99], traditional tools work well when there are no options at all, or there are options but very little uncertainty, or the decision is clear.

3.1.2 Decision Trees

3.1.2.1 Information about the Method

The decision trees analysis (DTA) tries to overcome the weaknesses of the NPV method. It accounts for uncertainty by the assignment of probabilities to each cash flow. This is a major difference to the NPV technique because there every cash flow has the probability of 100%. Moreover, managerial flexibility is considered as the DTA allows for managerial decisions and the selection of the best alternative subject to the decision. The DTA uses a tree consisting of squares and circles to describe the whole investment. The squares represent actions or decisions which are made by the management not at the time of valuation but at a later date when more information is available. On the other hand, circles show outcomes revealed by fate and not subject to managerial decisions [BrMy03], [CoWs05], and [ScMa03].

The following example eases the explanation of the decision tree method: a company considers investing 70,000 EUR in an Enterprise Application Integration (EAI) software. As their experience in the tricky implementation of such a tool is very moderate they might fail with a probability of 20%. If the EAI tool is installed properly the company receives 90,000 EUR and the opportunity arises to build a tool for customer and supplier administration, which is based on the EAI software. The company has the choice between two tools which differ in their initial investment (tool A < tool B) and in their earnings (tool A < tool B). In addition, the earnings depend on the business activity of the company. Concerning the activity three scenarios are possible: a) market conditions improve and the firm enlarges its business, b) market conditions do not change and neither the activity does, and c) market conditions deteriorate and thus the activity decreases. The comprehensive representation of this situation in a decision tree looks as follows:

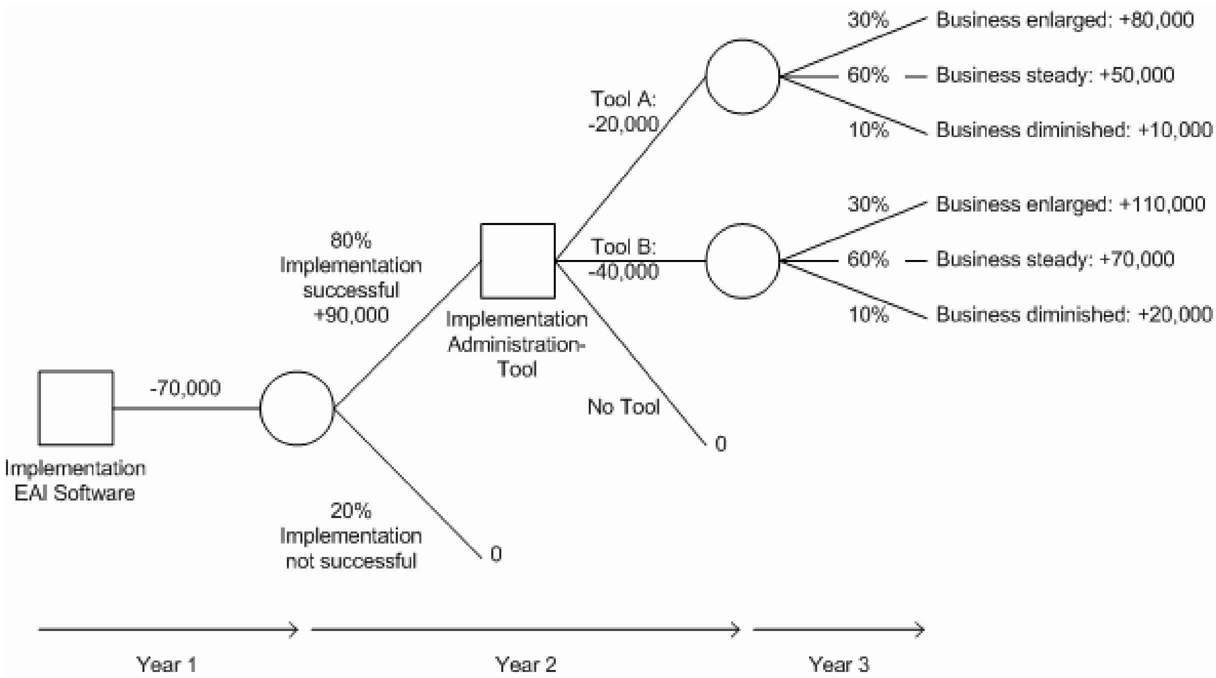


Figure 16: Decision tree example
Source: Own representation

In order to solve the decision tree it is necessary to go through the whole tree starting at its right end. For that it is necessary to discount all cash flows back with regard to the probability of each cash flow. Hence, for tool A and tool B the NPV at time t = 2 is given by (discount rate equals 6%):

$$\text{NPV}_2 \text{ Tool A} = \text{-}20{,}000 + \frac{0.3 * 80{,}000 + 0.6 * 50{,}000 + 0.1 * 10{,}000}{(1+0.06)^1} = 31{,}887$$

$$\text{NPV}_2 \text{ Tool B} = \text{-}40{,}000 + \frac{0.3 * 110{,}000 + 0.6 * 70{,}000 + 0.1 * 20{,}000}{(1+0.06)^1} = 32{,}642$$

The decision between the implementation of Tool A, Tool B, or no implementation is indicated in the tree by the square "Implementation Administration Tool". Its mathematical representation is the following:

Max (NPV Alternative 1, NPV Alternative 2, NPV Alternative 3)
= Max (31,887; 32,642; 0)
= 32,642

Thus, the management should implement Tool B and thereby receives 32,642 EUR. After this decision, is solved the next step is again to discount all probability-weighted cash flows.

$$\text{NPV}_0 \text{ EAI SW} = \text{-}70{,}000 + \frac{0.8 * (90{,}000 + 32{,}642) + 0.2 * 0}{(1+0.06)^1} = 22.560$$

The result shows a positive NPV for the investment, i.e. the implementation of the EAI software should be carried out.

3.1.2.2 Pros and Cons

The decision tree analysis improves the net present value method in two ways. First, it assigns probabilities to the cash flows which enable a better consideration of risk. Second, it considers managerial flexibility and thus allows for alternatives in the decisions. The latter issue is the more important improvement because now managers think about their possibilities and thus discover different options which increase the original NPV. Unfortunately, the DTA faces two big problems:

1. The tree soon becomes too complex
2. One single discount rate is used in the whole tree

The first disadvantage of the DTA is that the tree easily becomes an unmanageable “decision bush analysis”. In real investment settings the number of different paths expands geometrically with the number of decisions, outcome variables, or states considered of each variable [TrLe00]. Hence, it is important to show only the most important links between today’s and tomorrow’s decisions [BrMy03]. However, a more serious problem arises with the discount rate. As it can be seen in the example from above the DTA uses one common discount rate for the whole tree. This might be adequate in situations where the possibilities of two paths are 50% respectively but in other situations this does not hold. Moreover, options represented by managerial decision, reduce the risk of the investment. In the former example the management has the option but not the obligation to implement an administration tool. This opportunity should reduce the risk or respectively the discount rate in order to increase the NPV [CoAn02]. There exist some attempts in adjusting the discount rate but beside the growing complexity the question which always arises is “Which rate is really the correct one?” [TrLe00].

To summarise, the decision tree analysis makes steps in the right direction but cannot offer a correct valuation. Thus, there is a need for an approach which is able to calculate the value of options in an accurate way.

3.2 Valuation of Financial Options

The following section describes the fundamentals of financial option pricing. Unfortunately, it is not possible to apply these methods directly to real investments like the implementation of an integrative information system. Options of such a real world project are calculated with the real options valuation method which is introduced in the next section. Nevertheless, as the real options technique has its fundamentals in the principles and techniques of financial options theory, it is necessary to have a look at this one first.

3.2.1 Definitions and Notation

In finance, an **option** is a contract where one party (the holder or buyer) has the right but not the obligation to exercise a feature of the contract (the option) on or before a specified future date (the exercise date, expiry or maturity date). The other party (the writer or seller) has the obligation to honour the specified feature of the contract [Wiki06a]. More precisely, a **call option** gives its holder the right but not the obligation to purchase a share of stock in the underlying company at a fixed price (the exercise price or strike price) on or before the maturity date. On the other hand, a **put option** gives its holder the right but not the obligation to sell a share of stock at a fixed price on or before the maturity date. If the option can be exercised only on the maturity date then this type is called **European option**. Otherwise, if it can be exercised at any date up to maturity it is called **American option** [CoWS05].

As already mentioned, the owner only obtains the right but not the obligation to exercise. Thus, the main advantage of options is their ability to cut losses by simply not exercising them if things go wrong. On the other hand, there is no reduction of the achievable profits. The following position diagrams depict the option's payoff (option's exercise price = $ 55):

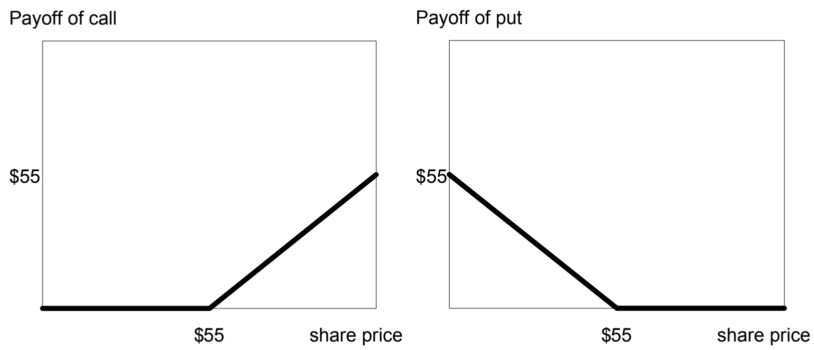


Figure 17: Position diagrams for call and put options
Source: adapted from [BrMy03]

Literature refers to such payoffs, i.e. payoffs which depend on the value of some other assets, as contingent payoffs. Position diagrams are not profit diagrams because they only show the payoffs on the maturity date. Profit diagrams account for the initial cost of buying the option (call option) or the initial proceeds from selling it (put option):

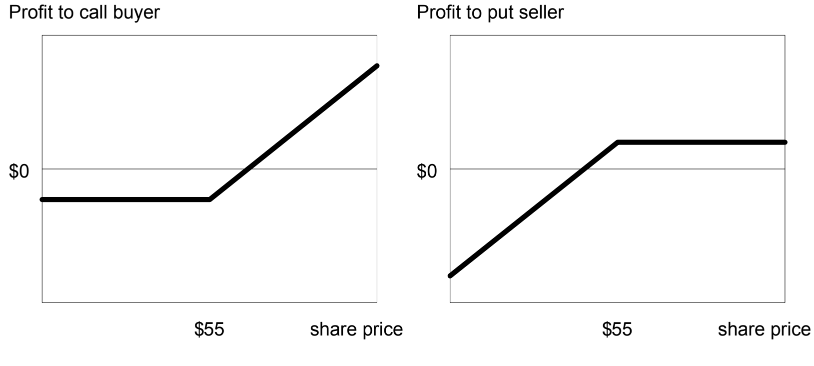


Figure 18: Profit diagrams for call and put options
Source: adapted from [BrMy03]

As put options are irrelevant for the present research, the study focuses on call options. This is explained in detail in section 3.3. A sufficient explanation for the time being is that the implementation of an integrative information system enables the company for follow-on opportunities which use the integration platform. Thus, such an integration platform can be regarded as a call option on follow-on opportunities.

Before the introduction of different models of option pricing some general issues are of interest. The following figure helps to understand a few basic principles:

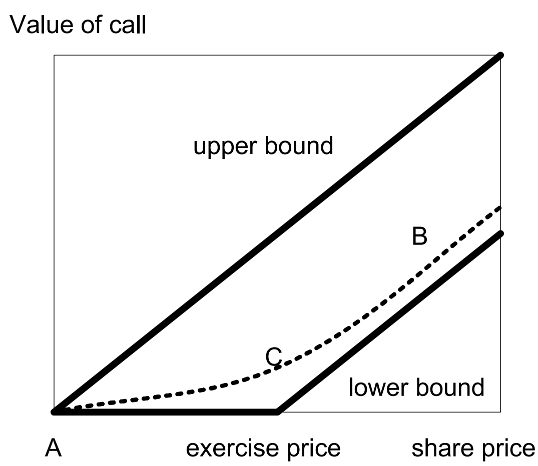


Figure 19: Lower and upper bound for the value of a call option
Source: [BrMy03]

The lower bound of the value of a call option is represented by its payoff stemming from an immediate exercise. Contrary, the upper bound is defined by the share price because the option's value cannot exceed it. As a result, the real value always lies between these two bounds. Like in point A the option is worthless if the stock is worthless. A stock price of zero means that investors do not expect any future value. On the other hand, if the stock price is large (point B), the option price becomes nearly the stock price less the present value of the exercise price. The higher the price of the share, the higher the probability that the option will be exercised. If the price is high enough then it is almost certain that an exercise of the option will take place. Thus, the line representing the call's value becomes parallel to the line expressing the lower bound. Point C shows the case when the stock price and the exercise price are equal. The option's value is not zero but always (not only in point C) above its lower bound because there is still the chance that the stock price increases until maturity.

Moreover, the difference between the option value and the lower bound is worth to pay attention to. The question arises which factors are influencing the value or price of a call option (C), i.e. the option premium. The following factors are necessary to determine the option's value [CoWs05]:

1. the price of the underlying asset (S),
2. the strike price (K),
3. the instantaneous variance of the returns of the underlying asset (σ^2),
4. the time to maturity (τ), and
5. the risk-free rate of interest (r_f).

While the first two points may be obvious, the others may not. A higher price of the underlying asset leads to a more valuable option whereas the lower the exercise price, the greater the value of the call option. A higher variance, i.e. more uncertainty affects the option's price positively. In such a case it is more probable that the stock price will exceed the exercise price. The situation for the time to maturity is quite similar because options with a later expiry date have a higher chance to climb above the exercise price. Consequently, more time to maturity leads to a more valuable option. The last parameter, the risk-free rate, is the least intuitive one. It is a result of the Black-Scholes model which is introduced in section 3.2.5. The option price according to the Black-Scholes model does not depend on individual risk preferences and is also independent of the expected return of the underlying asset. To summarise, the next table shows the relevant factors and how they influence the value of an option.

If there is an increase in …	**The change in the call option's price is:**
Stock price (S)	Positive (the underlying is more valuable)
Exercise price (K)	Negative (the cheaper the better)
Volatility of stock price (σ)	Positive (higher volatility leads to higher uncertainty and higher value)
Time to expiration (τ)	Positive (the later the higher the uncertainty, i.e. the range of possible outcomes and thus the value)
Risk free interest rate (r_f)	Positive (by exercising the option later than now the payment is deferred and the money can be invested otherwise)

Table 4: Main parameters of the price of an option
Source: adapted from [BrMy03] and [CoWS05]

Given the notation from above and the already introduced diagrams, the profit or loss at the maturity date (T) is given by the difference between the pay-off and the price (C) at time T:

- pay-off at time T = max $\{S_T – K, 0\} = (S_T – K)^+$
- price at the contract date t = $C (S_t, t)$

  price at time T (with $\tau = T – t$) = $C (S_t, t) e^{r\tau}$
- profit or loss at time T = $(S_T – K)^+ - C (S_t, t) e^{r\tau}$

The difference between the strike price (K) and the market price of the underlying (S) is also called the intrinsic value of an option. If it is greater than zero then the option is said to be in-the-money, if it is equal to zero then the option is at-the-money, and if it is less than zero then the option is out-of-the-money. However, the most difficult value to calculate, but also the most interesting one is the price of the option (C). The next sections introduce some models which help to solve this issue.

3.2.2 Principle of no Arbitrage and Perfect Financial Market

Each of the following valuation models is grounded on the principle of no arbitrage. An arbitrage opportunity arises if it is possible to make a profit for sure without any risk. In an ideal financial market all investors share the same information and are able to react instantaneously. Under such conditions an arbitrage opportunity disappears immediately: the investors try to realize the riskless profit right away which results in a change of the price [FrHH04]. This leads to the **law of one price**, i.e. two investments which have the same (present value) payoff must have the same price or arbitrage is possible [RoSh03]. An important conclusion is that when two investments have the same price at time T, their price must equal also at time t (for any t < T).

Besides this most important assumption, others are made and summarized in the concept of a **perfect financial market**: there are no arbitrage opportunities, no transaction costs, no taxes, and no restrictions on short selling. Lending rates equal borrowing rates and all securities are perfectly divisible [FrHH04].

3.2.3 Mathematical Background

For the explanation of the different valuation models some mathematical background is necessary. The following sections concentrate only on issues which are relevant for this study and are based on [RoSh03] as well as [FrHH04] where also more detailed information is available.

3.2.3.1 Fundamentals of Probability Theory

The universal sample space U refers to the set of all possible outcomes of an experiment:

$$U = \{1, 2, \dots, m\}.$$

Each outcome has the probability p_i which means that i is the outcome of the experiment:

$$p_i >= 0, \qquad i = 1, \dots, m, \quad \text{and} \quad \sum_{i=1}^{m} p_i = 1$$

An event A is a subset of U and occurs whenever the experiment's outcome is a point in A:

$$P(A) = \sum_{i \in A} p_i$$

The union of the events A and B ($A \cup B$) contains all outcomes that are in A, or in B, or in both. On the other hand, the intersection of A and B (AB or $A \cap B$) consists of all outcomes that are in A and B. The probability of the occurrence of A and B is calculated as follows:

$$P(A \cup B) = P(A) + P(B) - P(AB)$$ *"addition theorem of probability"*

Before it is possible to define P(AB) the conditional probability P(A|B) has to. It refers to the probability of A given that B has occurred:

$$P(A \mid B) = \frac{P(AB)}{P(B)}$$

Thus, for the intersection the following can be obtained:

$$P(AB) = P(A|B)\, P(B)$$ *"multiplication theorem of probability"*

If A and B are mutually exclusive or disjoint, i.e. AB = {}, then P(A|B) = P(A) and P(AB) = P(A) P(B).

Random variables are numerical quantities whose values are determined by the outcome of the experiment. An example may be the sum of heads that result of a series of coin flips. A random variable X has the possible values $x_1, x_2, \ldots, x_n$. The **probability distribution** of X is given by the following set of probabilities:

$$P\{X = x_j\} \qquad j = 1, \ldots, n \qquad \sum_{j=1}^{n} P\{X = x_j\} = 1$$

Thus, a random variable is not a variable but a function that assigns numbers to events. The expected value, or the expectation, or the mean of X is denoted by E[X]:

$$E[X] = \sum_{j=1}^{n} x_j P\{X = x_j\}$$

Hence, E[X] indicates the average of the possible values of X. The variance measures its spread:

$$Var(X) = E[(X - E[X])^2]$$

The square root of the variance is called the standard deviation.

A special random variable is the **Bernoulli random variable**. It refers to a random variable X which is equal to 1 with probability p and equal to 0 with probability 1 – p:

$$E[X] = 1\ p + 0\ (1 - p) = p$$
$$Var(X) = (1 - p)^2\ p + (0 - p)^2\ (1-p) = p - p^2$$

In general, the random variables $X_1, \ldots, X_n$ are said to be independent if the probabilities of any subset of them are not changed by any information of the others. A covariance Cov(X, Y) of zero expresses such a situation.

$$Cov(X, Y) = E[(X - E[X])(Y - E[Y])] \text{ or}$$
$$Cov(X, Y) = E[XY] - E[X]\ E[Y]$$

There exist some useful rules for E[X], Var(X), and Cov(X,Y) which are depicted below. A proof of them is available in [RoSh03].

$$E[aX + b] = a\,E[X] + b$$

$$Var(aX + b) = a^2\,Var(X)$$

$$Cov(X, Y) = Cov(Y, X)$$

$$Cov(X, X) = Var(X)$$

$$Cov(X_1 + X_2, Y) = Cov(X_1, Y) + Cov(X_2, Y)$$

$$Cov(cX, Y) = c\,Cov(X, Y)$$

$$Cov(c, Y) = 0$$

$$E\left[\sum_{j=1}^{k} X_j\right] = \sum_{j=1}^{k} E[X_j]$$

$$Var\left(\sum_{j=1}^{k} X_j\right) = \sum_{j=1}^{k} Var(X_j) + \sum_{i=1}^{k} \sum_{j \neq i} Cov(X_i, X_j)$$

$$Var\left(\sum_{j=1}^{k} X_j\right) = \sum_{j=1}^{k} Var(X_j) \quad \text{for independent random variables only}$$

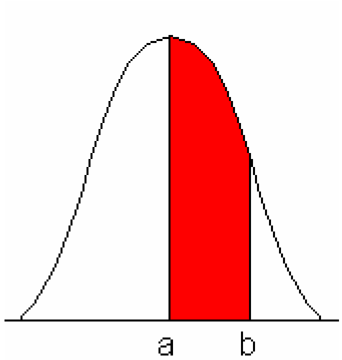
The possible values of the random variables are either **discrete** or **continuous**. Discrete random variables attain their values from a certain finite or countable set like the Bernoulli random variable. Continuous ones can take on any value within some interval where some of these values may be very likely and others not.

The **cumulative distribution function** F(x) describes the probability distribution of X:

$$F(x) = P(X \leq x) \qquad -\infty \leq x \leq +\infty$$

A continuous random variable X is illustrated additionally by the so called **probability density function** p(x), i.e. the probability that X equals to x. As a result, the probability that X assumes a value between a and b is the area under p between a and b:

$$P(a \le X \le b) = \int_a^b p(x)\,dx$$



As a result, F(x) for continuous random variables is given by

$$F(x) = \int_{-\infty}^{x} p(y)\,dy$$

The pendant of a probability density function for discrete random variables is the **probability mass function** which describes the probability that the random variable is equal to some value:

$$p(x) = \begin{cases} P\{X = x\} \text{ if } x \in U \\ 0 \text{ if } x \in R \backslash U \end{cases}$$

Normal random variables are a very important type of continuous random variables. Their probability density function is determined by two parameters, namely μ and σ:

$$\varphi_{\mu,\sigma^2}(x) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{(x-\mu^2)}{2\sigma^2}}$$

Such a random variable X is called N(μ, σ^2)-distributed. A plot of φ_{μ,σ^2} (x) yields a bell-shaped curve that is symmetric about the value μ with a variability measured by σ. Thus, μ represents the expected value E[X] and σ^2 the variance Var(X). If μ equals zero and σ equals one the random variable Z is called **standard normal random variable**:

$$\varphi(x) = \varphi_{0,1}(x) = \frac{1}{\sqrt{2\pi}} e^{-\frac{x^2}{2}}$$

A standard normal variable Z lies 68.26% of the time within σ, 95.44% of the time within 2σ, and 99,74% of the time within 3σ. Its cumulative distribution function is defined by:

$$\phi(x) = P(Z \le x) = \int_{-\infty}^{x} \varphi(y)\,dy \qquad \text{with } \phi(-x) = 1 - \phi(x) \text{ and } P(a < Z \le b) = \phi(b) - \phi(a)$$

It is possible to transform every normal random variable X into a standard normal one Z:

$$F(x) = \phi\left(\frac{x-\mu}{\sigma}\right) \qquad \text{with } \varphi_{\mu,\sigma^2}(x) = \frac{1}{\sigma}\varphi\left(\frac{x-\mu}{\sigma}\right) \text{ as } Z = \frac{X-\mu}{\sigma}$$

The random variable Y is called a **lognormal random variable** if

$$\ln(Y) = X \text{ or } Y = e^X$$

where X is a normal random variable. Thus, ln(Y) is a normal random variable with parameters μ and σ or ln(Y) is $N(\mu,\sigma^2)$-distributed. The cumulative distribution function and the probability density function look as follows:

$$F(x) = P(\ln X \le \ln x) = \phi\left(\frac{\ln x - \mu}{\sigma}\right) \qquad p(x) = \frac{1}{\sqrt{2\pi\sigma^2}}\frac{1}{x} e^{-\frac{(\ln x - \mu^2)}{2\sigma^2}} = \frac{1}{\sigma x}\varphi\left(\frac{\ln x - \mu}{\sigma}\right)$$

The expected value and the variance of a lognormal random variable Y are

$$E[Y] = e^{\mu + \sigma^2/2} \qquad \text{and} \qquad Var(Y) = e^{2\mu+2\sigma^2} - e^{2\mu+\sigma^2} = e^{2\mu+\sigma^2}(e^{\sigma} - 1)$$

The presumably most important theoretical result in probability theory is the **Central Limit Theorem**. It states that the sum of a large number of independent random variables which all have the same probability distribution is itself approximately a normal random variable.

$$S_n = \sum_{i=1}^{n} X_i \quad \text{for large n, } S_n \text{ is approximately } N(n\mu, n\sigma^2)\text{-distributed}$$

The sum of n Bernoulli random variables is binomially distributed, B(n,p). Hence, **binomial random variables** are described using two parameters: p and n, where n refers to the number of summands. In case of a Bernoulli random variable n is equal to one, i.e. it is B(1,p)-distributed. The expected value and the variance of a binomial random variable are:

$$E[X] = n\,p$$
$$Var(X) = n\,(p - p^2) = n\,p\,(1-p)$$

The probability that the binomial random variable X equals k, which means that it has k successes, is given by its probability mass function f(k; n, p):

$$P(X = k) = f(k; n, p) = \frac{n!}{k!\,(n-k)!}\, p^k\, (1-p)^{n-k}$$

If n is large enough then a B(n,p)-distributed random variable X can approximately be replaced by a N(np, np(1-p))-distributed random variable Z.

3.2.3.2 Stochastic Processes in Discrete Time

A stochastic process is an indexed collection of random variables $\{X_t,\ t \geq 0\}$. The index t represents the time and thus arranges the individual random variables. For reasons of simplicity the starting point of the process is always at t = 0. If t only takes integer values t = 0, 1, 2, …, then the process is said to be in discrete time. An example for such a process would be the daily collected stock price.

A **simple random walk** or **Bernoulli process** is a discrete stochastic process where the increments $I_t = X_t - X_{t-1}$ only take the values +1 (with probability p) or -1 (with probability 1-p). The random variables X_t are independent and identically distributed (i.i.d.).

$$X_t = X_0 + \sum_{k=1}^{t} I_k \qquad t = 1, 2, \dots \qquad P(I_k = 1) = p,\ P(I_k = -1) = 1-p \text{ (for all k)}$$

If p equals 0.5 then the random walk is symmetric, i.e. for $X_0 = 0$ the expected value $E[X_t] = 0$. A random walk with a positive drift ($p > ½$) or with a negative one ($p < ½$) has an expected value of $E[X_t] = t\,(2p - 1)$ as $E[I_t] = 1 * p + (-1) * (1 - p) = 2p - 1$.

A **binomial process** is quite similar to the Bernoulli process but has other increments than ±1. The increment of an upward movement is u and the increment of a downward one is d:

$$P(I_k = u) = p,\ P\ (I_k = -d) = 1\text{-}p$$

$$X_t = X_0 + nu - md \qquad n + m = t,\ n \geq 0,\ m \geq 0$$

The expected value and the variance of a binomial process are given by:

$$E[I_k] = up + (-d)(1\text{-}p) = up - d + dp = (u + d)\ p - d$$

$$E[X_t] = E[X_0] + t\ \{(u + d)\ p - d\}$$

$$Var(X_t) = t\ (u + d)^2\ p\ (1 - p)$$

Whereas $E[X_t]$ seems to be quite intuitive a closer look at $Var(X_t)$ is necessary. As the individual increments are independent and identically distributed the definition of the variance is as follows:

$$Var(X_t) = Var\ (X_0) + t\ Var(I_1)$$

In order to be more concrete let $Y_k = 1$ if $I_k = u$ and $Y_k = 0$ if $I_k = -d$:

$$Y_k = \frac{I_k + d}{u + d}$$

Moreover, B_t is the corresponding B(t,p)-distributed random variable:

$$B_t = \sum_{k=1}^{t} Y_k$$

Thus, it is possible to write for I_k and X_t

$$I_k = (u + d)\,Y_k - d$$
$$X_t = X_0 + (u + d)\,B_t - td.$$

According to section 3.2.3.1 $Var(B_t) = t\,p\,(1 - p)$ and $Var(aX + b) = a^2\,Var(X)$ which leads to

$$Var(X_t) = Var(X_0) + (u + d)^2\,Var(B_t) - 0 \text{ and}$$
$$Var(X_t) = t\,(u + d)^2\,p\,(1 - p).$$

For large t the distribution of a binomial process can be approximated by

$$B_t \approx N\,(t\,\{(u + d)\,p - d\},\; t\,(u + d)^2\,p\,(1 - p)).$$

Consequently, for $p = ½$ and $u = d = \Delta x$ the approximation from above leads to a distribution of $N\,(0, t\,(\Delta x)^2)$.

If the increment I_k can take a finite or countably infinite number of values or values out of a continuous set this is called a **general random walk**.

$$X_t = X_0 + \sum_{k=1}^{t} I_k \qquad t = 1, 2, \ldots \qquad I_1, I_2, \ldots \text{ i.i.d.}$$

As a result of the central limit theorem the following approximation works for all random walks if t is large enough and if $I_1, I_2, \ldots$ are i.i.d.:

$$X_t \approx N(t\,E[I_1],\; t\,Var(I_1))$$

Random walks are processes with independent increments, which means that I_{t+1} is independent of the past values $X_0, \ldots, X_t$. According to this characteristic they are also Markov-processes, i.e. if X_t is known then additional information about $X_0, \ldots, X_{t-1}$ does not change the estimate of X_{t+1}.

Given a **geometric random walk** not the absolute changes are stable over time but the relative or proportional ones are:

$$R_t = \frac{X_t}{X_{t-1}} \qquad t = 1, 2, \ldots \qquad R_1, R_2, \ldots \text{ i.i.d.}$$

$$X_t = R_t\, X_{t-1} = X_0 \prod_{k=1}^{t} R_k$$

$$P(R_k = u) = p,\ P(R_k = d) = 1 - p$$

$$E[R_k] = up + d\,(1\text{-}p) = up + d - dp = (u - d)p + d$$

$$E[X_t] = E[X_0] * (E[R_1])^t = E[X_0] * ((u - d)p + d)^t$$

Contrary, the process $\tilde{X}_t = \ln X_t$ is a non-geometric binomial process with starting value $\ln(X_0)$:

$$\ln X_t = \ln X_0 + \sum_{k=1}^{t} \ln R_k$$

$$I_k = \ln R_k$$

$$P(I_k = \ln u) = p,\ P(I_k = \ln d) = 1 - p$$

Thus, for large t, $\tilde{X}_t$ is approximately normally distributed and $X_t = \exp(\tilde{X}_t)$ is approximately lognormally distributed.

The increments I_k of a simple random walk are independent from the state X_t, i.e. every I_k has the same distribution. In case of a geometric random walk the increment R_k depends on the current state X_{t-1}. This is clearly a better approximation but not a sufficient one for a model of a stock price evolution based on the current price level. For that purpose **binomial processes with state dependent (and possibly time dependent) increments** are more suitable:

$$X_t = X_{t-1} + I_t \qquad t = 1, 2, \ldots$$

$$P(I_t = u) = p\,(X_{t-1}, t),\ \ P(I_t = -d) = 1 - p\,(X_{t-1}, t)$$

The function p(x, t) associates a probability to every possible value at every time t. As a result the increments are not equal any more but the process is still a Markov-process. Moreover, a **geometric binomial process with state dependent increments** is defined by:

$$X_t = R_t X_{t-1} = X_0 \prod_{k=1}^{t} R_k$$

$$P(R_k = u) = p(X_{t-1}, t),\ P(R_k = d) = 1 - p(X_{t-1}, t)$$

3.2.3.3 The Stock Price as a Stochastic Process

Although the stock price of a security is a process in discrete time, it is approximated by a process in continuous time as calculations for this one are easier. However, discrete models are more intuitive and so they are very useful in simulations.

The previous chapter defines a simple symmetric random walk $\{X_n;\ n \geq 0\}$ with $X_0 = 0$, i.i.d. increments $I_n = \pm 1$, and $p = 0.5$. If this process is accelerated, the period of time between two observations becomes smaller and correspondingly the increments too. The outcome is a stochastic process $\{X_t^{\Delta};\ t \geq 0\}$ in continuous time with t representing a positive real number. The process increases or decreases in time step Δt with probability 0.5 by Δx. Consequently, at time $t = n\,\Delta t$ the process equals

$$X_t^{\Delta} = \sum_{k=1}^{t} I_k \Delta x = X_n \Delta x \qquad \text{where } I_k = \pm 1 \text{ with } p = 0.5$$

$$E[X_t^{\Delta}] = 0$$

$$\begin{aligned} Var(X_t^{\Delta}) &= Var(X_n \Delta x) \\ &= (\Delta x)^2\, Var(X_n) \\ &= (\Delta x)^2 \{n\,(u + d)^2\, p\,(1 - p)\} \\ &= (\Delta x)^2 \{n * 2^2 * 0.5 * 0.5\} \\ &= n\,(\Delta x)^2 \\ &= t * (\Delta x)^2 / \Delta t \end{aligned}$$

As a next step Δt and Δx become smaller whereas $Var(X_t^{\Delta})$ should be finite and not converge to 0 to keep the process random:

$$\Delta t \to 0,\ \Delta x = c\sqrt{\Delta t}\text{, such that } Var(X_t^{\Delta}) \to c^2 t$$

If Δt is small then n is large and the simple symmetric random walk X_n is approximately normally distributed. As a result for all t (not only for t = n Δt) the distribution of X_t^Δ is given by:

$$X_t^\Delta \approx N(0, n(\Delta x)^2) \approx N(0, c^2 t)$$

The limiting process $\{X_t;\ t \geq 0\}$ which is obtained from $\{X_t^\Delta;\ t \geq 0\}$ for $\Delta t \to 0$ and $\Delta x = c\sqrt{\Delta t}$ has 3 major properties:

1. X_t is $N(0, c^2 t)$ distributed for $t \geq 0$
2. $\{X_t^\Delta;\ t \geq 0\}$ has independent increments, i.e. $X_t - X_s$ is independent of X_s as the random walk X_n has independent increments ($0 \leq s < t$)
3. the increment $X_t - X_s$ is $N(0, c^2 (t-s))$ distributed

A stochastic process in continuous time which satisfies these three properties is called **Wiener process or Brownian motion** with starting point $X_0 = 0$. In case of a **standard Wiener process** $\{W_t;\ t \geq 0\}$ c equals 1 and henceforth $\Delta x = \sqrt{\Delta t}$. This leads to the following ($0 \leq s < t$):

$$E[W_t] = 0$$

$$Var(W_t) = t$$

$$\begin{aligned}
Cov(W_t, W_s) &= Cov(W_t - W_s + W_s, W_s) \\
&= Cov(W_t - W_s, W_s) + Cov(W_s, W_s) \\
&= 0 + Var(W_s) \qquad \text{(0 because of property two)} \\
&= s
\end{aligned}$$

The Wiener process W_t has no jumps but fluctuates heavily. As its increments are independent it is a Markov process, i.e. W_t depends only on W_s and the increment $W_t - W_s$. Given $W_s = x$ and property three, W_t is $N(x, t-s)$ distributed:

$$P(a < W_t < b \mid W_s = x) = \int_a^b \varphi_{\mu,\sigma^2}(y)dy \text{ with } \varphi_{\mu,\sigma^2}(x) = \frac{1}{\sigma}\varphi(\frac{x-\mu}{\sigma}) = \frac{1}{\sqrt{t-s}}\varphi(\frac{y-x}{\sqrt{t-s}})$$

$$P(a < W_t < b \mid W_s = x) = \int_a^b \frac{1}{\sqrt{t-s}}\varphi(\frac{y-x}{\sqrt{t-s}})\, dy$$

If not a simple symmetric random walk is considered, but one with a drift its expected value is not zero but $E[X_n] = n\,(2p - 1)$. The stochastic process changes as follows:

$$E[X_t^\Delta] = E[X_n]\,\Delta x = n\,(2p-1)\,\Delta x = (2p-1)\,t\,\Delta x/\Delta t$$

$$\begin{aligned} Var(X_t^\Delta) &= Var\,(X_n\,\Delta x) \\ &= (\Delta x)^2\, Var\,(X_n) \\ &= (\Delta x)^2\,\{n\,(u+d)^2\,p\,(1-p)\} \\ &= (\Delta x)^2\,\{n * 2^2 * p * (1-p)\} \\ &= n\,4p\,(1-p)\,(\Delta x)^2 \\ &= 4p\,(1-p)\,t\,(\Delta x)^2/\Delta t \end{aligned}$$

For $\Delta t \to 0$, $\Delta x = \sqrt{\Delta t}$, and $p = 0.5\,(1 + \mu\sqrt{\Delta t})$ it holds for all t

$$E[X_t^\Delta] \to \mu t,\ Var(X_t^\Delta) \to t$$

This limiting process $\{X_t;\ t \geq 0\}$ describes a **Wiener process with drift or trend μt** and is a result of the standard Wiener process W_t:

$$X_t = \mu t + W_t$$

The next step in the development of a stochastic model of the stock price is the introduction of the Itô-integral as it is not possible to apply the common Riemann-integral to stochastic processes. However, the Riemann-integral is constructed as follows to calculate the area under the non-negative real-valued function f(x) of the interval [a, b]. The interval [a, b] is divided into n intervals $[x_{i-1}, x_i]$ with length Δx_i. If $n \to \infty$ then $\Delta x_i \to 0$ and for any $\xi_I \in [x_{i-1}, x_i]$ the integral is defined by

$$\int_a^b f(x)dx = \lim_{n\to\infty} \sum_{i=1}^{n} f(\xi_i)\Delta x_i$$

Thus, the area under f(x) is approximated by many small rectangles with height $f(\xi_I)$ and width Δx_i. A major issue is that ξ_I can take any value of the interval $[x_{i-1}, x_i]$, which means that it does not matter whether ξ_I is at the left or at the right border.

For the integration of a stochastic process $\{Y_t, t \geq 0\}$it is assumed that the process is not anticipating, i.e. the process up to time s does not contain any information about future increments $W_t - W_s$ $(t > s)$ of the Wiener process. In other words, Y_s is independent of $W_t - W_s$. The **Itô-integral** with respect to a Wiener process is not defined as above as the limit of the sum of the suitably weighted function but as the limit of the sum of the randomly weighted random function:

$$\int_0^t Y_s \, dW_s = \lim_{n\to\infty} Z_n$$

$$Z_n = \sum_{k=1}^{n} Y_{(k-1)\Delta t} \left(W_{k\Delta t} - W_{(k-1)\Delta t}\right) \qquad \Delta t = t/n$$

Consequently, Z_n is the product of two independent random variables,

1. the process at the left and only the left border of the interval $[(k-1)\Delta t, k\Delta t]$ and
2. the increment of the Wiener process in this interval.

In comparison to the Riemann-integral the calculation of the Itô-integral is different:

$$\int_0^t f_s \, df_s = \frac{1}{2}\left(f_t^2 - f_0^2\right)$$

$$\int_0^t W_s \, dW_s = \frac{1}{2}\left(W_t^2 - W_0^2\right) - \frac{t}{2} = \frac{1}{2}\left(W_t^2 - t\right) \qquad \text{as } W_0^2 = 0$$

A proof of the Itô-integral is observable below ($n\,\Delta t = t$):

$$\begin{aligned}\frac{1}{2}(W_t^2 - W_0^2) &= \frac{1}{2}\sum_{k=1}^{n}(W_{k\Delta t}^2 - W_{(k-1)\Delta t}^2)\\ &= \frac{1}{2}\sum_{k=1}^{n}(W_{k\Delta t} - W_{(k-1)\Delta t})(W_{k\Delta t} + W_{(k-1)\Delta t})\\ &= \frac{1}{2}\sum_{k=1}^{n}(W_{k\Delta t} - W_{(k-1)\Delta t})^2 + \sum_{k=1}^{n}(W_{k\Delta t} - W_{(k-1)\Delta t})\,W_{(k-1)\Delta t}\end{aligned}$$

The second term converges to $\int_0^t W_s\, dW_s$ and the first one is a sum of n i.i.d. random variables. Due to the law of large numbers it can be approximated by its expected value: $\frac{n}{2}E[(W_{k\Delta t} - W_{(k-1)\Delta t})^2] = \frac{n}{2}(\Delta x)^2 = \frac{n}{2}(\sqrt{\Delta t})^2 = \frac{t}{2}$. Hence, the equation from above can be modified into

$$\frac{1}{2}(W_t^2 - W_0^2) = \frac{t}{2} + \int_0^t W_s\, dW_s$$

which shows exactly the proof of Itô's integral.

Based on the already introduced Wiener process with drift μt and $N(\mu t, t)$ distribution the **generalized Wiener process with drift rate μ and variance σ^2** is defined by:

$$X_t = \mu t + \sigma W_t \qquad t \geq 0,\ N(\mu t, \sigma^2 t)$$

Applying the Itô-Integral leads to

$$X_t = X_0 + \int_0^t \mu\, ds + \int_0^t \sigma\, dW_s\,.$$

For an explanation it is helpful to start with a definition of the increment of X_t in a small time interval Δt:

$$X_{t+\Delta t} - X_t = \mu\,\Delta t + \sigma(W_{t+\Delta t} - W_t) \text{ or } \frac{X_{t+\Delta t} - X_t}{\Delta t} = \mu + \sigma\frac{W_{t+\Delta t} - W_t}{\Delta t}$$

For $\Delta t \to 0$ the differential notation yields

$$\frac{dX_t}{dt} = \mu + \sigma\frac{dW_t}{dt}$$
$$dX_t = \mu\,dt + \sigma\,dW_t$$

and the integral form results in

$$X_t = X_0 + \int_0^t \mu\,ds + \int_0^t \sigma\,dW_s \qquad \text{where} \int_0^t dW_s = W_t - W_0 = W_t$$

If μ and σ are not constant but dependent on the time and the state of the process then dX_t is defined by:

$$dX_t = \mu(X_t, t)\,dt + \sigma(X_t, t)\,dW_t$$
$$X_t = X_0 + \int_0^t \mu(X_s, s)\,ds + \int_0^t \sigma(X_s, s)\,dW_s$$

Such a process, defined as the solution of a stochastic differential equation is called **Itô-process**. As the increments of the Wiener process between time t and s do not depend on the events up to time t the process is still Markovian.

All in all there are two possibilities to model the stock price. On the one hand the generalized Wiener process and, on the other hand, the Itô-process. The Wiener process has two major disadvantages:

1. it allows for negative stock prices
2. the probability of the absolute change in the price does not depend on the present price as the price difference has the same normal distribution no matter what the price is

The second point stipulates that the probability of a price movement from EUR 20 to EUR 15 (loss of 25%) is the same as a movement from EUR 10 to EUR 5 (loss of 50%). Hence, the Itô-process is used to model the stock price S_t:

$$dS_t = \mu(S_t, t)\, dt + \sigma(S_t, t)\, dW_t$$

A simplification of this formula by substituting the unknown functions $\mu(S_t, t)$ and $\sigma(S_t, t)$ by two unknown parameters μ and σ results in:

$$\frac{E[dS_t]}{S_t} = \frac{\mu(S_t, t)\, dt}{S_t} + \frac{\sigma(S_t, t)\, E[dW_t]}{S_t}$$

$$\frac{E[S_{t+dt} - S_t]}{S_t} = \mu\, dt \quad \text{for } \mu(S_t, t) = \mu\, S_t \quad \text{and} \quad E[dW_t] = 0$$

An interpretation of the equation from above is that the rate of return is proportional to the investment horizon dt and dependent only on the relative difference and not on the absolute one. Analogously, $\sigma(S_t, t) = \sigma\, S_t$ accounts for the fact that the absolute size of the stock price change is proportional to the currency unit in which the stock is quoted.

$$dS_t = \mu\, S_t\, dt + \sigma\, S_t\, dW_t$$

$$\frac{dS_t}{S_t} = \mu\, dt + \sigma\, dW_t$$

Such a process is called **geometric Brownian motion** and can be approximated by a geometric random walk. Modelling the stock price in this way overcomes the major shortcoming of the generalized Wiener Process: it assumes that it is the percentage change in price, and not the absolute change, which probability does not depend on the present price.

Finally, introductory section ends with the explanation of **Itô's Lemma**. It represents the chain rule for stochastic processes as the ordinary chain rule for differential equations is not applicable for them:

$$
\begin{aligned}
Y_t &= g(S_t, t) \\
dY_t &= dg(S_t, t) \\
&= g(S_{t+dt}, t+dt) - g(S_t, t) \\
&= \left(\frac{\partial g}{\partial S} \mu(S_t, t) + \frac{1}{2} \frac{\partial^2 g}{\partial S^2} \sigma^2(S_t, t) + \frac{\partial g}{\partial t} \right) dt + \frac{\partial g}{\partial S} \sigma(S_t, t) dW
\end{aligned}
$$

For a heuristic derivation of Itô's Lemma one uses the Taylor series to second order about the point (S_t, t) [Wiki06b]:

$$
g(S_{t+dt}, t+dt) \approx g(S_t, t) + \frac{\partial g}{\partial S} dS_t + \frac{\partial g}{\partial t} dt + \frac{1}{2} \left(\frac{\partial^2 g}{\partial S^2} (dS_t)^2 + 2 \frac{\partial^2 g}{\partial X \partial t} dX_t dt + \frac{\partial^2 g}{\partial t^2} (dt)^2 \right)
$$

Here the last two terms are ignored for $dt \to 0$. Putting this result in the formula from above yields:

$$
dY_t = \frac{\partial g}{\partial S} dS_t + \frac{\partial g}{\partial t} dt + \frac{1}{2} \frac{\partial^2 g}{\partial S^2} (dS_t)^2
$$

As a next step follows the simplification of $(dS_t)^2$:

$$
\begin{aligned}
(dS_t)^2 &= (\mu(S_t, t)\, dt + \sigma(S_t, t)\, dW_t)^2 \\
&= \mu^2(S_t, t)\, (dt)^2 + 2\, \mu(S_t, t)\, \sigma(S_t, t)\, dt\, dW_t + \sigma^2(S_t, t)\, (dW_t)^2
\end{aligned}
$$

The first term is of size $(dt)^2$ and the second one of $dt\sqrt{dt}$. It is possible to neglect them as they are smaller than dt. Moreover, $E[(dW_t)^2] = dt$ and $dW_t = W_{t+dt} - W_t$ is of the size of its standard deviation $\sqrt{dt}$. Hence, the third term is of size dt and this leads to

$$
(dS_t)^2 = \sigma^2(S_t, t)\, dt\,.
$$

To be more precise, it can be shown that $(dW_t)^2 = dt$ for $dt \to 0$. For a more detailed depiction the interested reader is referred to e.g. [JiLi05, p. 61]. With this simplification of $(dS_t)^2$ it is easy to obtain dY_t:

$$
\begin{aligned}
dY_t &= \left(\frac{\partial g}{\partial t} + \frac{1}{2} \frac{\partial^2 g}{\partial S^2} \sigma^2(S_t, t) \right) dt + \frac{\partial g}{\partial S} dS_t \\
&= \left(\frac{\partial g}{\partial t} + \frac{1}{2} \frac{\partial^2 g}{\partial S^2} \sigma^2(S_t, t) \right) dt + \frac{\partial g}{\partial S} \left(\mu(S_t, t)\, dt + \sigma(S_t, t)\, dW_t \right) \\
&= \left(\frac{\partial g}{\partial S} \mu(S_t, t) + \frac{1}{2} \frac{\partial^2 g}{\partial S^2} \sigma^2(S_t, t) + \frac{\partial g}{\partial t} \right) dt + \frac{\partial g}{\partial S} \sigma(S_t, t)\, dW_t
\end{aligned}
$$

The last line shows exactly the desired result from above.

3.2.4 Overview

Based on the initial study of Black and Scholes in 1973 many valuation models for options were developed. This thesis concentrates on the following major studies for the pricing of European call options:

Type of option	**Author**	**Reference**
Option on a stock	Black and Scholes	[BlSc73]
Option on a stock which pays dividends	Merton	[MeRo73]
Option on a stock with an uncertain exercise price	Fischer	[FiSt78]
Exchange options on an asset which pays no dividends	Margrabe	[MaWi78]
Compound options	Geske	[GeRo79a]

Table 5: Valuation models for European call options
Source: Own representation

The following sections will provide detailed information about these models. Each of these continuous-time models offers a closed form solution, i.e. it is possible to calculate the value of the option using one formula only. Alternatively, Cox, Ross, and Rubinstein [CoRR79] present a discrete-time model, namely the binomial tree model.

If an American call option is considered the situation is quite easy if the underlying asset pays no dividends. Hence, the closed form solutions can be applied directly without any modification. Otherwise, i.e. if the underlying asset pays dividends, the valuation via a closed form solution becomes more difficult. In such cases the application of numerical procedures like the binomial tree simplifies the calculation to a great extent. A more detailed discussion is available in section 3.2.10.

3.2.5 European Call Options

3.2.5.1 General

The most important valuation model for financial options was derived by Black and Scholes [BlSc73]. They developed a formula for the value of an European option on a stock which pays no dividends. Beside a perfect financial market with no arbitrage opportunity they assume the following:

- the short-term interest rate is known and constant through time
- the underlying stock price S_t follows a geometric Brownian motion

  $$\frac{dS_t}{S_t} = \mu\, dt + \sigma\, dW_t\,,$$

  where the expected rate of return μ and the standard deviation σ are constant (for further explanation see 3.2.3.3, geometric Brownian motion, where $\mu\,(S_t, t)$ was substituted by $\mu\, S_t$ and $\sigma(S_t, t)$ by $\sigma\, S_t$)

The main idea is the construction of a riskless portfolio which consists of one share of the underlying stock long and an appropriate amount of options short. These two positions are combined in an offsetting manner into a hedge position. As a result, the value of this position is independent of fluctuations in the underlying asset and so this position earns a risk-free rate of return. Moreover, the portfolio is not permanently riskless but only for an infinitesimally short period of time. To keep the portfolio riskless it is necessary to frequently adjust it [AmKu99]. However, the fact that the value does not depend on the risk preference of the individual investor was the real breakthrough of the study. The 1997 Nobel economics price was awarded to M. Scholes and R. Merton (F. Black had died) for this formula and a series of contributions based on it [JiLi05].

The original study is split into two parts: in the first part they develop a mathematical model for the pricing of options (the “Black-Scholes partial differential equation”) and the second part presents the pricing formula as solution of this equation (the “Black-Scholes formula”).

3.2.5.2 Black-Scholes Partial Differential Equation

As already mentioned, the first step of Black and Scholes [BlSc73] is the construction of a riskless portfolio P. This portfolio consists of the underlying share S long and x call options C short:

$$P = S - x\,C$$

The amount x of the call options has to be chosen in a way that the portfolio's return between t and t + dt equals the risk free interest rate r_f.

$$\frac{P_{t+dt} - P_t}{P_t} = r_f\ dt$$
$$dS_t - x\ dC_t = (S_t - x\ C_t)\ r_f\ dt$$

However, for an explanation it is more comprehensible to follow Jiang [JiLi05] who builds the riskless portfolio in the exactly opposite way, namely of one call option long and Δ shares short:

$$P = C - \Delta S$$
$$\frac{P_{t+dt} - P_t}{P_t} = r_f\ dt$$
$$dC_t - \Delta\ dS_t = (C_t - \Delta\ S_t)\ r_f\ dt$$

As S_t follows a geometric Brownian motion ($dS_t = \mu\, S_t\ dt + \sigma\, S_t\ dW_t$) the following intermediate result is obtained:

$$dC_t - \Delta(\mu\, S_t\ dt + \sigma\, S_t\ dW_t) = (C_t - \Delta\, S_t)\ r_f\ dt$$

Moreover, $C_t = C(S_t, t)$ where S_t follows a geometric Brownian motion. Hence, Itô's Lemma is applicable including a substitution of $\mu\,(S_t, t)$ by $\mu\, S_t$ and of $\sigma(S_t, t)$ by $\sigma\, S_t$:

$$dC_t = \left(\frac{\partial C}{\partial S}\mu S + \frac{1}{2}\frac{\partial^2 C}{\partial S^2}\sigma^2 S^2 + \frac{\partial C}{\partial t}\right)dt + \frac{\partial C}{\partial S}\sigma S\, dW_t$$

This formula is used together with the intermediate result and leads to the subsequent outcome:

$$\left(\frac{\partial C}{\partial S}\mu S + \frac{1}{2}\frac{\partial^2 C}{\partial S^2}\sigma^2 S^2 + \frac{\partial C}{\partial t}\right)dt + \frac{\partial C}{\partial S}\sigma S\, dW_t - \Delta\,\mu\, S\, dt - \Delta\,\sigma\, S\, dW_t = (C - \Delta S)\, r_f\ dt$$

$$\left(\frac{\partial C}{\partial S}\mu S + \frac{1}{2}\frac{\partial^2 C}{\partial S^2}\sigma^2 S^2 + \frac{\partial C}{\partial t} - \Delta\,\mu\, S\right)dt + \left(\frac{\partial C}{\partial S}\sigma S - \Delta\,\sigma\, S\right) dW_t = (C - \Delta S)\, r_f\ dt$$

As the portfolio has to be risk-free it is necessary to choose Δ in a way that the risk (dW_t) disappears:

$$\Delta = \frac{\partial C}{\partial S}$$

$$\left(\frac{\partial C}{\partial S}\mu S + \frac{1}{2}\frac{\partial^2 C}{\partial S^2}\sigma^2 S^2 + \frac{\partial C}{\partial t} - \frac{\partial C}{\partial S}\mu\, S\right)dt = \left(C - \frac{\partial C}{\partial S} S\right) r_f\ dt$$

$$\frac{1}{2}\frac{\partial^2 C}{\partial S^2}\sigma^2 S^2 + \frac{\partial C}{\partial t} = \left(C - \frac{\partial C}{\partial S} S\right) r_f$$

$$\frac{\partial C}{\partial t} + \frac{1}{2}\sigma^2 S^2 \frac{\partial^2 C}{\partial S^2} + r_f S \frac{\partial C}{\partial S} - r_f C = 0$$

This is the Black-Scholes equation that describes the option price movement. It is valid for any financial instrument (e.g. a forward contract) contingent on an underlying with price S_t if S_t follows a geometric Brownian motion. Additionally the price of the financial instrument must be a function of only the price S_t and time: $C_t = C(S_t, t)$ [FrHH04]. However, this study focuses on its application on call options.

3.2.5.3 Black-Scholes Formula

The Black-Scholes equation has many solutions which are prior to boundary conditions. These conditions are characterised by the type of the financial instrument under consideration. For an European Call option these boundary conditions are given by [FrHH04]:

1. $C(S_t, t) = \max\{S_T - K, 0\}$, $0 < S < \infty$

   The call option is only exercised if the stock price is greater than the strike price.

2. $C(0,t) = 0,\ 0 \le t \le T$

   If the geometric Brownian Motion S_t equals 0 at any time t then it will be zero at T (the process is absorbed by zero).

3. $\lim_{S \to \infty} C(S_t, t) - S_T = 0,\ 0 \le t \le T$

   If the stock price is much larger than the strike price ($S_t \rangle\rangle K$) at any time t then $S_T \rangle\rangle K$ will hold with a very high probability. The result would be a cash flow of $S_T - K \approx S_T$.

Subject to these conditions the solution of the PDE is as follows:

$$C(S,\tau) = S\,\phi(d_1) - Ke^{-r_f \tau}\phi(d_2) \quad \text{or}$$
$$C(S,t) = S\,N(d_1) - Ke^{-r_f (T-t)}N(d_2)$$

In the formula $\phi(x)$ and $N(x)$ denote the cumulative distribution function of the standard normal random variable Z:

$$N(x) = \phi(x) = P(Z \le x) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{x} e^{-\frac{u^2}{2}}\, du$$

The parameters used are defined by

$$d_1 = \frac{\ln \frac{S}{K} + (r_f + \frac{1}{2}\sigma^2)(T-t)}{\sigma\sqrt{T-t}} \quad \text{and}$$
$$d_2 = d_1 - \sigma\sqrt{T-t}.$$

An interpretation of the Black-Scholes formula is that the price of a call option equals the discounted expected value of $\max(S_T - K, 0)$ [ZiHe03]:

$$C(S,t) = e^{-r_f \tau}E[\max(S_T - K, 0)]$$
$$C(S,t) = e^{-r_f \tau}E[S_T - K \mid S_T > K]$$
$$C(S,t) = e^{-r_f \tau}E[S_T \mid S_T > K] - e^{-r_f \tau}E[K \mid S_T > K]$$

Whereas the first part in this equation refers to the discounted expected earnings of the option, the second part deal with the discounted expected costs. Setting

$$E[S_T \mid S_T > K] = Se^{-r_f\tau} N(d_1) \text{ and}$$
$$E[K \mid S_T > K] = K\,N(d_2)$$

then leads to the Black-Scholes formula.

3.2.5.4 Proof

The exact way of receiving the Black-Scholes formula by solving the partial differential equation is available in [JiLi05], [FrHH04], or directly in [BlSc73]. Instead of this issue for the present thesis a proof of the solution is more interesting consisting of two parts: the fulfilment of the partial differential equation itself and the fulfilment of the boundary conditions [KrLu05]. The first step is to determine some derivatives:

$$\begin{aligned}
\frac{\partial C}{\partial S} &= \frac{\partial\left(S\,N(d_1) - Ke^{-r(T\text{-}t)}N(d_2)\right)}{\partial S} \\
&= N(d_1) + S\frac{\partial N(d_1)}{\partial S} - Ke^{-r(T\text{-}t)}\frac{\partial N(d_2)}{\partial S} \\
&= N(d_1) + S\,N'(d_1)\frac{\partial d_1}{\partial S} - Ke^{-r(T\text{-}t)}N'(d_2)\frac{\partial d_2}{\partial S}
\end{aligned}$$

$$\frac{\partial d_1}{\partial S} = \frac{\partial \dfrac{\ln\frac{S}{K} + (r + \frac{1}{2}\sigma^2)(T-t)}{\sigma\sqrt{T-t}}}{\partial S} = \frac{\dfrac{\partial \ln\frac{S}{K}}{\partial S}\sigma\sqrt{T-t}}{\left(\sigma\sqrt{T-t}\right)^2} = \frac{\frac{K}{S}\frac{K}{K^2}}{\sigma\sqrt{T-t}} = \frac{1}{S\sigma\sqrt{T-t}}$$

$$\frac{\partial d_2}{\partial S} = \frac{1}{S\sigma\sqrt{T-t}}$$

Next, it is shown that $S\,N'(d_1) = Ke^{-r(T-t)}N'(d_2)$:

$$\frac{N'(d_1)}{N'(d_2)} = \frac{\frac{1}{\sqrt{2\pi}} e^{-0.5\,d_1{}^2}}{\frac{1}{\sqrt{2\pi}} e^{-0.5\,d_2{}^2}} = e^{0.5\,d_2{}^2 - 0.5\,d_1{}^2} = e^{0.5\,(d_1 - \sigma\sqrt{T-t})^2 - 0.5\,d_1{}^2}$$

$$= e^{-0.5 \cdot 2\,d_1 \sigma\sqrt{T-t} + 0.5\,\sigma^2 (T-t)}$$

$$= e^{-\left(\frac{\ln\frac{S}{K} + (r+0.5\,\sigma^2)(T-t)}{\sigma\sqrt{T-t}}\right)\sigma\sqrt{T-t} + 0.5\,\sigma^2(T-t)}$$

$$= e^{-\ln\frac{S}{K} - r\,(T-t)} = \frac{e^{r\,(T-t)}}{e^{\ln\frac{S}{K}}} = \frac{e^{r\,(T-t)}}{\frac{S}{K}} = \frac{K\,e^{r\,(T-t)}}{S}$$

As a result, it is possible to give a simple expression for $\frac{\partial C}{\partial S}$:

$$\frac{\partial C}{\partial S} = N(d_1) + S\,N'(d_1)\frac{\partial d_1}{\partial S} - Ke^{-r(T-t)}N'(d_2)\frac{\partial d_2}{\partial S}$$

$$= N(d_1) + S\,N'(d_1)\frac{1}{S\,\sigma\sqrt{T-t}} - Ke^{-r(T-t)}N'(d_2)\frac{1}{S\,\sigma\sqrt{T-t}}$$

$$= N(d_1) + S\,N'(d_1)\frac{1}{S\,\sigma\sqrt{T-t}} - S\,N'(d_1)\frac{1}{S\,\sigma\sqrt{T-t}}$$

$$= N(d_1)$$

Next follows the determination of $\frac{\partial^2 C}{\partial S^2}$:

$$\frac{\partial^2 C}{\partial S^2} = \frac{\partial N(d_1)}{\partial S} = N'(d_1)\frac{\partial d_1}{\partial S} = N'(d_1)\frac{1}{S\,\sigma\sqrt{T-t}}$$

Additionally, the subsequent derivative is needed:

$$
\begin{aligned}
\frac{\partial C}{\partial t} &= \frac{\partial\left(S\,N(d_1) - Ke^{-r(T-t)}N(d_2)\right)}{\partial t} \\
&= S\frac{\partial N(d_1)}{\partial t} - \frac{Ke^{-r(T-t)}}{\partial t}N(d_2) - Ke^{-r(T-t)}\frac{\partial N(d_2)}{\partial t} \\
&= S\,N'(d_1)\frac{\partial d_1}{\partial t} - K\frac{e^{-r(T-t)}}{\partial t}N(d_2) - Ke^{-r(T-t)}N'(d_2)\frac{\partial d_2}{\partial S} \\
&= S\,N'(d_1)\frac{\partial d_1}{\partial t} - Ke^{-r(T-t)}\,r\,N(d_2) - Ke^{-r(T-t)}N'(d_2)\frac{\partial d_2}{\partial S} \\
&= Ke^{-r(T-t)}\,N'(d_2)\frac{\partial d_1}{\partial t} - Ke^{-r(T-t)}\,r\,N(d_2) - Ke^{-r(T-t)}N'(d_2)\frac{\partial d_2}{\partial S} \\
&= Ke^{-r(T-t)}\left(N'(d_2)\frac{\partial d_1}{\partial t} - r\,N(d_2) - N'(d_2)\frac{\partial d_2}{\partial S}\right) \\
&= Ke^{-r(T-t)}\left(\left(\frac{\partial d_1}{\partial t} - \frac{\partial d_2}{\partial S}\right)N'(d_2) - r\,N(d_2)\right)
\end{aligned}
$$

For further simplification it is necessary to calculate

$$
\begin{aligned}
\frac{\partial d_1}{\partial t} &= \frac{\partial\dfrac{\ln\frac{S}{K} + (r+\frac{1}{2}\sigma^2)(T-t)}{\sigma\sqrt{T-t}}}{\partial t} = \frac{\dfrac{\partial\,(r+\frac{1}{2}\sigma^2)(T-t)}{\partial t}\sigma\sqrt{T-t} - \left(\ln\frac{S}{K} + (r+\frac{1}{2}\sigma^2)(T-t)\right)\dfrac{\sigma\sqrt{T-t}}{\partial t}}{\left(\sigma\sqrt{T-t}\right)^2} \\
&= \frac{-(r+\frac{1}{2}\sigma^2)\sigma\sqrt{T-t} - \left(\ln\frac{S}{K} + (r+\frac{1}{2}\sigma^2)(T-t)\right)\dfrac{-\sigma}{2\sqrt{T-t}}}{\sigma^2(T-t)} \\
&= \frac{-(r+\frac{1}{2}\sigma^2)(T-t)^{-0.5} + \left(\ln\frac{S}{K} + (r+\frac{1}{2}\sigma^2)(T-t)\right)\frac{1}{2}(T-t)^{-1.5}}{\sigma} \\
&= \frac{-(r+\frac{1}{2}\sigma^2)(T-t)^{-0.5} + \ln\frac{S}{K}\frac{1}{2}(T-t)^{-1.5} + (r+\frac{1}{2}\sigma^2)\frac{1}{2}(T-t)^{-0.5}}{\sigma} \\
&= \frac{0.5\ln\frac{S}{K}}{\sigma}(T-t)^{-1.5} + \frac{-0.5\,(r+0.5\sigma^2)}{\sigma}(T-t)^{-0.5}
\end{aligned}
$$

,

$$\frac{\partial d_2}{\partial t} = \frac{\partial d_2}{\partial t} - \frac{\partial(\sigma\sqrt{T-t})}{\partial t}$$

$$= \frac{0.5\ln\frac{S}{K}}{\sigma}(T-t)^{-1.5} + \frac{-0.5\,(r+0.5\sigma^2)}{\sigma}(T-t)^{-0.5} + 0.5\sigma(T-t)^{-0.5}$$

$$= \frac{0.5\ln\frac{S}{K}}{\sigma}(T-t)^{-1.5} + \frac{-0.5\,(r-0.5\sigma^2)}{\sigma}(T-t)^{-0.5}$$

This results in the following expression:

$$\frac{\partial d_1}{\partial t} - \frac{\partial d_2}{\partial t} = \frac{-0.5\,(r+0.5\sigma^2)}{\sigma}(T-t)^{-0.5} - \frac{-0.5\,(r-0.5\sigma^2)}{\sigma}(T-t)^{-0.5} = \frac{-\sigma}{2\sqrt{T-t}}$$

Consequently, $\frac{\partial C}{\partial t}$ is given by:

$$\frac{\partial C}{\partial t} = Ke^{-r(T-t)}\left(\left(\frac{\partial d_1}{\partial t} - \frac{\partial d_2}{\partial S}\right)N'(d_2) - r\,N(d_2)\right) = Ke^{-r(T-t)}\left(\frac{-\sigma}{2\sqrt{T-t}}N'(d_2) - r\,N(d_2)\right)$$

For a proof of the partial differential equation one has to insert the formula and its derivatives into it:

$$\frac{\partial C}{\partial t} + \frac{1}{2}\sigma^2 S^2\frac{\partial^2 C}{\partial S^2} + rS\frac{\partial C}{\partial S} - rC = 0$$

$$Ke^{-r(T-t)}\left(\frac{-\sigma}{2\sqrt{T-t}}N'(d_2) - r\,N(d_2)\right) + \frac{1}{2}\sigma^2 S^2 N'(d_1)\frac{1}{S\sigma\sqrt{T-t}} + rSN(d_1) - rS\,N(d_1) + rKe^{-r(T-t)}N(d_2) = 0$$

$$Ke^{-r(T-t)}\frac{-\sigma}{2\sqrt{T-t}}N'(d_2) + \frac{1}{2}\sigma^2 S^2 N'(d_1)\frac{1}{S\sigma\sqrt{T-t}} = 0$$

$$Ke^{-r(T-t)}\frac{-\sigma}{2\sqrt{T-t}}N'(d_1 - \sigma\sqrt{T-t}) + \sigma SN'(d_1)\frac{1}{2\sqrt{T-t}} = 0$$

$$-Ke^{-r(T-t)}\,N'(d_1 - \sigma\sqrt{T-t}) + SN'(d_1) = 0$$

Using $N'(x) = \frac{1}{\sqrt{2\pi}}e^{-\frac{x^2}{2}}$ for further transformation yields to:

$$-Ke^{-r(T-t)}N'(d_1-\sigma\sqrt{T-t})+SN'(d_1)=0$$

$$-Ke^{-r(T-t)}\frac{1}{\sqrt{2\pi}}e^{-0.5(d_1-\sigma\sqrt{T-t})^2}+S\frac{1}{\sqrt{2\pi}}e^{-0.5d_1^2}=0$$

$$-Ke^{-r(T-t)}\frac{1}{\sqrt{2\pi}}e^{-0.5d_1^2}e^{d_1\sigma\sqrt{T-t}}e^{-0.5\sigma^2(T-t)}+S\frac{1}{\sqrt{2\pi}}e^{-0.5d_1^2}=0$$

$$-Ke^{-r(T-t)}e^{d_1\sigma\sqrt{T-t}}e^{-0.5\sigma^2(T-t)}+S=0$$

Finally, the definition of d_1, namely $d_1=\frac{\ln\frac{S}{K}+(r+\frac{1}{2}\sigma^2)(T-t)}{\sigma\sqrt{T-t}}$, leads to:

$$-Ke^{-r(T-t)}\frac{S}{K}e^{r(T-t)}e^{0.5\sigma^2(T-t)}e^{-0.5\sigma^2(T-t)}+S=0$$

$$-S+S=0$$

After this proof of the fulfilment of the partial differential equation the proof of the fulfilment of the boundary conditions is still open. As the conditions two and three are a result of the Brownian motion it is enough only to consider condition one [KrLu05]:

Boundary condition one: $C(S_t, t) = \max\{S_T - K, 0\}, 0 < S < \infty$

The first step is to calculate the following limits:

$$\lim_{t\to T} d_1 = \lim_{t\to T}\frac{\ln\frac{S}{K}+(r+0.5\sigma^2)(T-t)}{\sigma\sqrt{T-t}} = \lim_{t\to T}\frac{\ln\frac{S}{K}}{\sigma\sqrt{T-t}}+\lim_{t\to T}\frac{r+0.5\sigma^2}{\sigma}\sqrt{T-t} = \begin{cases} +\infty \text{ if } S>K \\ 0 \text{ if } S=K \\ -\infty \text{ if } S<K \end{cases}$$

$$\lim_{t\to T} d_2 = \lim_{t\to T} d_1 - \lim_{t\to T}\sigma\sqrt{T-t} = \lim_{t\to T} d_1$$

$$\lim_{t\to T} N(d_1) = \lim_{t\to T} N(d_2) = \begin{cases} 1 \text{ if } S>K \\ 0.5 \text{ if } S=K \\ 0 \text{ if } S<K \end{cases}$$

$$\lim_{t\to T} e^{-r(T-t)} = 1$$

As a consequence $\lim_{t\to T} C(S_t, t)$ equals

$$\lim_{t\to T} C(S_t, t) = \lim_{t\to T} S\,N(d_1) - Ke^{-r(T-t)}N(d_2) \begin{cases} S-K & \text{if } S>K \\ 0.5\,S - 0.5\,K = 0 & \text{if } S=K \\ 0\,S - 0\,K = 0 & \text{if } S<K \end{cases},$$

which obviously fulfils the boundary condition under consideration.

3.2.6 European Call Options with Dividends

3.2.6.1 General

The break-through of Black and Scholes led to a high interest in the topic of financial option valuation. Merton [MeRo73] developed several extensions to the model including one for the pricing of European call options where the underlying pays dividends at a constant rate. Thus, the assumptions of Black and Scholes are still valid: a perfect market with no arbitrage opportunity, the model of the prices of the underlying asset follows a geometric Brownian motion, and the expected rate of return μ as well as the standard deviation σ is constant:

3.2.6.2 Modified Partial Differential Equation

In order to keep the approach equal to the previous section it makes sense to follow Jiang [JiLi05]. This implies a different notation than the one used by Merton, but the results are the same.

The already known Δ-hedging technique is used for constructing a riskless portfolio consisting of one call option long and Δ shares short. The shares pay dividends at the constant rate q (q = S / amount of the dividend) which leads to:

$$P = C - \Delta S$$

$$\frac{P_{t+dt} - P_t}{P_t} = r_f\ dt$$

$$(C_{t+dt} - \Delta\, S_{t+dt} - \Delta\, q\, S_t\ dt) - (C_t - \Delta\, S_t) = (C_t - \Delta\, S_t)\, r_f\ dt$$

$$dC_t - \Delta\, dS_t - \Delta\, q\, S_t\ dt = (C_t - \Delta\, S_t)\, r_f\ dt$$

Again S_t follows a geometric Brownian motion ($dS_t = \mu\, S_t\ dt + \sigma\, S_t\ dW_t$):

$$dC_t - \Delta(\mu\, S_t\ dt + \sigma\, S_t\ dW_t) - \Delta\, q\, S_t\ dt = (C_t - \Delta\, S_t)\, r_f\ dt$$

Like in the derivation of the Black-Scholes formula the use of Itô's Lemma for $C_t = C(S_t, t)$ is possible.

$$dC_t = \left(\frac{\partial C}{\partial S} \mu S + \frac{1}{2} \frac{\partial^2 C}{\partial S^2} \sigma^2 S^2 + \frac{\partial C}{\partial t} \right) dt + \frac{\partial C}{\partial S} \sigma S \, dW_t$$

$$\left(\frac{\partial C}{\partial S} \mu S + \frac{1}{2} \frac{\partial^2 C}{\partial S^2} \sigma^2 S^2 + \frac{\partial C}{\partial t} \right) dt + \frac{\partial C}{\partial S} \sigma S \, dW_t - \Delta \mu S \, dt - \Delta \sigma S \, dW_t - \Delta q S \, dt = (C - \Delta S) \, r_f \, dt$$

$$\left(\frac{\partial C}{\partial S} \mu S + \frac{1}{2} \frac{\partial^2 C}{\partial S^2} \sigma^2 S^2 + \frac{\partial C}{\partial t} - \Delta \mu S - \Delta q S \right) dt + \left(\frac{\partial C}{\partial S} \sigma S - \Delta \sigma S \right) dW_t = (C - \Delta S) \, r_f \, dt$$

The amount of shares Δ is chosen exactly in the same way as above to eliminate the risk which is represented by dW_t:

$$\Delta = \frac{\partial C}{\partial S}$$

$$\left(\frac{\partial C}{\partial S} \mu S + \frac{1}{2} \frac{\partial^2 C}{\partial S^2} \sigma^2 S^2 + \frac{\partial C}{\partial t} - \frac{\partial C}{\partial S} \mu S - \frac{\partial C}{\partial S} q S \right) dt = \left(C - \frac{\partial C}{\partial S} S \right) r_f \, dt$$

$$\frac{1}{2} \frac{\partial^2 C}{\partial S^2} \sigma^2 S^2 + \frac{\partial C}{\partial t} - \frac{\partial C}{\partial S} q S = \left(C - \frac{\partial C}{\partial S} S \right) r_f$$

$$\frac{\partial C}{\partial t} + \frac{1}{2} \sigma^2 S^2 \frac{\partial^2 C}{\partial S^2} + (r_f - q) S \frac{\partial C}{\partial S} - r_f C = 0$$

3.2.6.3 Modified Formula

The solution of the partial differential equation is subject to the same boundary conditions like the Black-Scholes formula and is as follows:

$$C(S, \tau) = S \, e^{-q\tau} \phi(d_1) - K e^{-r_f \tau} \phi(d_2) \quad \text{or}$$

$$C(S, t) = S e^{-q(T-t)} N(d_1) - K e^{-r_f (T-t)} N(d_2)$$

$$d_1 = \frac{\ln \frac{S}{K} + (r_f - q + \frac{1}{2} \sigma^2)(T - t)}{\sigma \sqrt{T - t}} \quad \text{and}$$

$$d_2 = d_1 - \sigma \sqrt{T - t}.$$

To summarise, the formula equals the Black-Scholes equation with one difference. The dividends are part of the share's yield and not receivable for the holder of the option. Hence, the dividends reduce the value of the option.

3.2.7 European Call Options with an Uncertain Exercise Price

3.2.7.1 General

Fischer [FiSt78] introduces another extension of the initial model of Black and Scholes [BlSc73]. In his study he accounts for options with an uncertain exercise price. So the assumptions from Black and Scholes (a perfect market with no arbitrage opportunity, the model of the prices of the underlying asset follows a geometric Brownian motion, and the expected rate of return μ as well as the standard deviation σ are constant) are extended by the postulation that not only the underlying stock but also the exercise price follows a geometric Brownian motion:

$$\frac{dS_t}{S_t} = \mu_S \, dt + \sigma_S \, dW_S \text{ and } \frac{dK_t}{K_t} = \mu_K \, dt + \sigma_K \, dW_K$$

While μ_S refers to the expected rate of return on the stock, μ_K denotes the expected rate of increase of the exercise price. On the other hand σ_S is the standard deviation of the return as well as σ_K represents the standard deviation of the increase. Moreover, the correlation between the Wiener processes dW_S and dW_K is ρ_{SK}.

3.2.7.2 Mathematical Problem and its Solution

Again the Δ-hedging technique is used to create a riskless portfolio. As already known it is possible to hedge the changes in the share's price by combining one call option long and Δ shares short. In addition, a second hedge security prevents against changes in the exercise price. As a result, the procedure is very similar to the one from above and leads to:

$$C(S,K,t) = S\,N(d_1) - Ke^{-(\mu_S - \mu_K)(T-t)} N(d_2)$$

$$d_1 = \frac{\ln\frac{S}{K} + (\mu_S - \mu_K + \frac{1}{2}\sigma^2)(T-t)}{\sigma\sqrt{T-t}}$$

$$d_2 = d_1 - \sigma\sqrt{T-t}.$$

$$\sigma^2 = \sigma_S^2 - 2 \cdot \sigma_S \cdot \sigma_K \cdot \rho_{SK} + \sigma_K^2$$

If the exercise price is constant then $\sigma_K = 0$, $\mu_K = 0$, and $\mu_S = r$. Hence, σ^2 becomes equal to σ_S^2 and C(S, K, t) reduces to the standard Black-Scholes formula.

3.2.8 European Exchange Options

3.2.8.1 General

Based on the preliminary study of Black and Scholes [BlSc73] and Merton's extensions [MeRo73] Margrabe developed a pricing equation for options to exchange one risky asset for another [MaWi78]. An example for an exchange option would be the option to buy yen with Australian dollars. A stock tender offer is also an example as an option to exchange shares in one stock for shares in another stock is obtained.

Margrabe's assumptions are similar to those from Black and Scholes, i.e. a perfect market with no arbitrage opportunity is supposed. Like the stock price in the Black-Scholes model the prices of asset one (U) and asset two (V) follow a geometric Brownian motion. Again the expected rate of return μ and the standard deviation σ are constant:

$$\frac{dU_t}{U_t} = \mu_U \, dt + \sigma_U \, dW_{U\,t} \text{ respectively } \frac{dV_t}{V_t} = \mu_V \, dt + \sigma_V \, dW_{V\,t}$$

The correlation between the Wiener processes dW_U and dW_V is ρ_{UV}.

3.2.8.2 Mathematical Problem and its Solution

The payoff from an European option to give up an asset worth U_T at time T and receive in return an asset worth V_T [HuJo00] is expressed by:

$$\max(V_T - U_T, 0)$$

Thus, an exchange option represents a call option on V_T with exercise price U_T [MaWi78]:

$$C(V_t, U_t, t) = \max(V_T - U_T, 0)$$

Moreover, the option is worth at least zero and not more than V_T:

$$0 \leq C(V_t, U_t, t) \leq V_T$$

Quite similar to Black and Scholes, Margrabe also builds a position P to eliminate risk:

$$P = C - x\,V - y\,U$$

Hence, the position is hedged by selling x units of V short and buying –y units of U. Margrabe defines x as $\frac{\partial C}{\partial V}$ and y equals $\frac{\partial C}{\partial U}$. Whereas in the Black-Scholes model the hedged position earns the risk free rate of interest, Margrabe's portfolio has no return. His weights which eliminate risk in the hedge position also make it costless.

$$\frac{P_{t+dt} - P_t}{P_t} = 0$$

$$dC_t - x\,dV_t - y\,dU_t = 0$$

By using Itô's Lemma Merton [MeRo73, p. 164] provides a solution for the change in the option price C if $C(V_t, U_t, t)$ is the option price function:

$$dC = \frac{\partial C}{\partial V}dV + \frac{\partial C}{\partial U}dU + z dt + 0.5\left(\frac{\partial^2 C}{\partial V^2}\sigma_V^2 V + 2\frac{\partial^2 C}{\partial V \partial U}\sigma_V \cdot \sigma_U \cdot \rho_{UV} \cdot V \cdot U + \frac{\partial^2 C}{\partial U^2}\sigma_U^2 U\right)dt,$$

where $z = \frac{\partial C}{\partial t}$. As $dC - x\,dV - y\,dU = 0$ it is possible to write

$$0 = z + 0.5\left(\frac{\partial^2 C}{\partial V^2}\sigma_V^2 V + 2\frac{\partial^2 C}{\partial V \partial U}\sigma_V \cdot \sigma_U \cdot \rho_{UV} \cdot V \cdot U + \frac{\partial^2 C}{\partial U^2}\sigma_U^2 U\right).$$

Subject to the conditions

$$C(V_t, U_t, t) = \max\,(V_T - U_T, 0) \text{ and } 0 \leq C(V_t, U_t, t) \leq V_T$$

the solution of the PDE from above is:

$$C(V, U, t) = V\,N(d_1) - U\,N(d_2)$$

$$d_1 = \frac{\ln\frac{V}{U} + \frac{1}{2}\sigma^2(T-t)}{\sigma\sqrt{T-t}} \quad \text{and}$$

$$d_2 = d_1 - \sigma\sqrt{T-t}.$$

$$\sigma^2 = \sigma_V^2 - 2\cdot\sigma_V\cdot\sigma_U\cdot\rho_{UV} + \sigma_U^2$$

Margrabe's result is a special case of the result of Fischer [FiSt78] and it is also possible to show that Margrabe's formula can be derived from Fischer's [KiMi95].

3.2.8.3 Proof

The easiest way to proof the result from above is a transformation into the Black-Scholes problem. For that purpose it is necessary to eliminate the risk of one asset by defining the price of asset V in terms of U:

$$C(V_t, U_t, t) = C(V_t / U_t, U_t / U_t, t) = C(V_t / U_t, 1, t)$$

$$C(V_t / U_t, 1, t) = \max(V_T / U_T - U_T / U_T, 0) = \max(V_T / U_T - 1, 0)$$

This equals a transformation into a simple call option with an exercise price equal to one. Additionally, the interest rate is zero as the hedged position does not earn the risk free rate but has no return at all. In this context Margrabe [MaWi78] argues, that a riskless loan denominated in units of asset two has an interest rate of zero in a perfect market. A lender of one unit of asset two demands one unit of asset two back as repayment of the principle. He charges no interest on the loan because the appreciation of asset two during the period of the loan is an equilibrium compensation for the investment and the risk.

All in all, this is a special case of the Black-Scholes problem (S = V/U, K = 1, r = 0):

$$C(S,K,t) = S\,N(d_1) - Ke^{-r(T-t)}N(d_2)$$

$$C(V/U,1,t) = V/U\,N(d_1) - 1e^{-0(T-t)}N(d_2)$$

$$C(V/U,1,t) = V/U\,N(d_1) - N(d_2)$$

$$C(V,U,t) = V\,N(d_1) - U\,N(d_2)$$

$$d_1 = \frac{\ln\frac{S}{K} + (r + \frac{1}{2}\sigma^2)(T-t)}{\sigma\sqrt{T-t}}$$
$$= \frac{\ln\frac{V}{U} + \frac{1}{2}\sigma^2(T-t)}{\sigma\sqrt{T-t}}$$
$$d_2 = d_1 - \sigma\sqrt{T-t}.$$

It remains the definition of σ which refers to the volatility of the underlying asset V/U:

$$\sigma^2 = \sigma_V^2 - 2 \cdot \sigma_V \cdot \sigma_U \cdot \rho_{UV} + \sigma_U^2 .$$

3.2.9 European Compound Options

3.2.9.1 General

In 1979 Geske [GeRo79a] introduced a valuation formula for compound options. Contrary to Black and Scholes he regards a call option not only as an option on a stock but as an option on an option: first it is an option on a stock but second the stock again is an option on the value of the firm. An example illustrates the explanation: a company has common stock and bonds outstanding. The company cannot issue any new claims on the firm, or pay cash dividends, or repurchase shares prior to the maturity of the bonds. In such a situation the bondholders own the company's assets and have given the stockholders the option to buy the assets back when the bond expires. Hence, the stock represents an option on the value of the firm with exercise price equal to the face value of the bonds.

Geske shows that the Black-Scholes model refers to a special case of the compound option model. The problem of the Black-Scholes formula is its assumption that the variance rate of the return on the stock is constant but with compound options it is not. Here the rate depends on the level of the stock price, respectively on the value of the firm (for a more detailed description see below). Beside this issue, Geske assumes the same as Black and Scholes:

- a perfect financial market with no arbitrage opportunity
- the short-term interest rate is known and constant through time

- the value of the firm V follows a geometric Brownian motion

  $$\frac{dV}{V} = \mu_V \, dt + \sigma_V \, dW_V,$$

  where the expected rate of return μ and the standard deviation σ are constant (for further explanation see 3.2.3.3, geometric Brownian motion, where $\mu(S_t, t)$ was substituted by μS_t and $\sigma(S_t, t)$ by σS_t)

3.2.9.2 Modified Partial Differential Equation

As already depicted a call on the firm's stock represents a compound option (V refers to the value of the firm):

$$C = f(S,t) = f(g(V,t),t)$$

The holder of C has the right to buy S at time T* with exercise price K. Afterwards he/she has then the right to buy V with exercise price M at time T (T* < T). Hence, S is an ordinary call option with strike price equal to M, the face value of the bonds. According to Black and Scholes, the proper partial differential equation is

$$\frac{\partial S}{\partial t} + \frac{1}{2}\sigma_V^2 V^2 \frac{\partial^2 S}{\partial V^2} + r_f V \frac{\partial S}{\partial V} - r_f S = 0.$$

The central boundary condition $S(V_t, t) = \max\{V_T - M, 0\}$ leads to:

$$S = g(V,t) = V\,N(d_1) - Me^{-r_f (T-t)} N(d_2)$$

$$d_1 = \frac{\ln\frac{V}{M} + (r_f + \frac{1}{2}\sigma_V^2)(T-t)}{\sigma_V \sqrt{T-t}}$$

$$d_2 = d_1 - \sigma_V \sqrt{T-t}.$$

Since the call option is a function of the value of the firm and time, its return is expressed by the following equation:

$$\frac{dC}{C} = \mu_C \, dt + \sigma_C \, dW_C$$

As a result of the functional relationship between C and V the parameters μ_C, σ_C, and dW_C are explicitly related to μ_V, σ_V, and dW_V. Moreover, the familiar partial differential equation from Black and Scholes leads to:

$$\frac{\partial C}{\partial t} + \frac{1}{2}\sigma_V^2 V^2 \frac{\partial^2 C}{\partial V^2} + r_f V \frac{\partial C}{\partial V} - r_f C = 0$$

3.2.9.3 Modified Formula

Unfortunately, the equation has no quick solution as the boundary condition $C(S_t, t) = \max\{S_{T^*} - K, 0\}$ poses a problem: S is not part of the differential equation but is itself defined via a partial differential equation (see above). Thus, when solving the partial differential equation for C, the one for S must hold also. Geske provides the following solution for this problem [GeRo79a]:

$$\begin{aligned} C &= f(g(V), t) \\ &= V\, B(h + \sigma_V \sqrt{\tau_1}, k + \sigma_V \sqrt{\tau_2}; \sqrt{\tau_1 / \tau_2}) - Me^{-r_f \tau_2} B(h, k; \sqrt{\tau_1 / \tau_2}) - Ke^{-r_f \tau_1} N(h) \end{aligned}$$

The time to maturity of the first option C equals τ_1 = T*-t whereas τ_2 = T-t identifies the time to maturity of the second option S with T* < T. The definition of the parameters h and k is as follows:

$$h = \frac{\ln \frac{V}{\overline{V}} + (r_f - \frac{1}{2}\sigma_V^2)\tau_1}{\sigma_V \sqrt{\tau_1}}$$

$$k = \frac{\ln \frac{V}{M} + (r_f - \frac{1}{2}\sigma_V^2)\tau_2}{\sigma_V \sqrt{\tau_2}}$$

While N() is the common cumulative standard normal distribution function, B(a, b, ρ) is a bivariate cumulative normal distribution function with upper integral limits a and b and correlation coefficient $\rho = \sqrt{\tau_1 / \tau_2}$:

$$B(a, b, \rho) = \int_{-\infty}^{a} \int_{-\infty}^{b} \frac{1}{2\pi\sqrt{1-\rho^2}} \exp\left(-0.5 \frac{x^2 - 2\rho xy + y^2}{1-\rho^2} \right) dx\, dy$$

$\overline{V}$ is the probability measure over the value of the firm, being that value of the firm which solves the integral equation $S_{T*} - K = 0$. Hence, $\overline{V}$ is that value of V such that

$$V\,N(d_1) - Me^{-r_f \tau_2} N(d_2) - K = 0 \text{ with } t = T^* \text{ and } \tau_2 = T\text{-}T^*.$$

For values of V less than $\overline{V}$ will be no exercise of the option but only for values greater than $\overline{V}$.

According to Geske, the Black-Scholes formula is a special case of his formula, namely when the call is written on the equity of an unlevered firm, i.e. when M = 0 or T = ∞. However, comparing Geske's formula to Black and Scholes' one can say that the Black-Scholes equation with the "proper" variance ($\sigma_S^2 \rightarrow \sigma_V^2$) and the "appropriate" stock price (S → V) yields the compound option equation:

$$\frac{\partial C}{\partial t} + \frac{1}{2}\sigma_V^2 V^2 \frac{\partial^2 C}{\partial V^2} + r_f V \frac{\partial C}{\partial V} - r_f C = 0 \text{ vs. } \frac{\partial C}{\partial t} + \frac{1}{2}\sigma_S^2 S^2 \frac{\partial^2 C}{\partial S^2} + r_f S \frac{\partial C}{\partial S} - r_f C = 0$$

This means not only S but also σ_S is a function of V and t and hence, σ_S is not constant any more. Instead of replacing $\sigma_S(S_t, t)$ by $\sigma_S\, S_t$, Geske uses $\sigma_S(S(V_t), t)$ [GeRo79a].

In order to provide assistance for the actual use of the formula Hull [HuJo02] mentions that Drezner provides a way of calculating B(a, b, ρ) to four-decimal-place accuracy [DrZv78]. The Visual Basic code of this method is available in the appendix and is adapted from [GlDe07]. Moreover, this thesis uses the Newton Raphson method [Wiki07b] for determining the critical value $\overline{V}$. Generally, this method modifies the variable x in several trials until f(x) = 0. The trial n+1 uses a better approximation of x than trial n by calculating $x_{n+1} = x_n - f(x_n) / f'(x_n)$. When applying this approach in the context of Geske's formula x equals $\overline{V}$, f(x) represents the Black-Scholes formula $V\,N(d_1) - Me^{-r_f \tau_2} N(d_2) - K = 0$, and $f'(x) = N(d_1)$ (see 3.2.5.4).

3.2.9.4 Extension: Dividend Paying Underlying

According to Hull [HuJo00] a small change in Geske's original formula is sufficient in order to account for an underlying which pays dividends at the constant rate q:

$$
\begin{aligned}
C &= f(g(V),t) \\
&= V\,e^{-q\tau_2}\,B(h+\sigma_V\sqrt{\tau_1},\,k+\sigma_V\sqrt{\tau_2};\sqrt{\tau_1/\tau_2}) - Me^{-r_f\tau_2}B(h,k;\sqrt{\tau_1/\tau_2}) - Ke^{-r_f\tau_1}N(h)
\end{aligned}
$$

$$h = \frac{\ln\frac{V}{\overline{V}} + (r_f - q - \frac{1}{2}\sigma_V^2)\tau_1}{\sigma_V\sqrt{\tau_1}}$$

$$k = \frac{\ln\frac{V}{M} + (r_f - q - \frac{1}{2}\sigma_V^2)\tau_2}{\sigma_V\sqrt{\tau_2}}$$

For the calculation of the critical value $\overline{V}$ via the Newton Raphson method the following derivative $\frac{\partial C}{\partial V}$ is necessary:

$$
\begin{aligned}
\frac{\partial C}{\partial V} &= \frac{\partial\left(Ve^{-q(T-t)}N(d_1) - Ke^{-r(T-t)}N(d_2)\right)}{\partial S} \\
&= e^{-q(T-t)}N(d_1) + Ve^{-q(T-t)}\frac{\partial N(d_1)}{\partial V} - Ke^{-r(T-t)}\frac{\partial N(d_2)}{\partial V} \\
&= e^{-q(T-t)}N(d_1) + V\,e^{-q(T-t)}N'(d_1)\frac{\partial d_1}{\partial V} - Ke^{-r(T-t)}N'(d_2)\frac{\partial d_2}{\partial V}
\end{aligned}
$$

$$\frac{\partial d_1}{\partial V} = \frac{\partial \dfrac{\ln\frac{V}{K} + (r - q + \frac{1}{2}\sigma^2)(T-t)}{\sigma\sqrt{T-t}}}{\partial V} = \frac{\frac{\partial \ln\frac{V}{K}}{\partial V}\sigma\sqrt{T-t}}{\left(\sigma\sqrt{T-t}\right)^2} = \frac{\frac{K}{V}\frac{K}{K^2}}{\sigma\sqrt{T-t}} = \frac{1}{V\,\sigma\sqrt{T-t}}$$

$$\frac{\partial d_2}{\partial V} = \frac{1}{V\,\sigma\sqrt{T-t}}$$

$$\frac{N'(d_1)}{N'(d_2)} = \frac{\frac{1}{\sqrt{2\pi}} e^{-0.5\, d_1^2}}{\frac{1}{\sqrt{2\pi}} e^{-0.5\, d_2^2}} = e^{0.5\, d_2^2 - 0.5\, d_1^2} = e^{0.5\,(d_1 - \sigma\sqrt{T-t})^2 - 0.5\, d_1^2}$$

$$= e^{-0.5 \cdot 2\, d_1 \sigma\sqrt{T-t} + 0.5\,\sigma^2 (T-t)}$$

$$= e^{-\left(\frac{\ln\frac{V}{K} + (r-q+0.5\,\sigma^2)(T-t)}{\sigma\sqrt{T-t}}\right)\sigma\sqrt{T-t} + 0.5\,\sigma^2(T-t)}$$

$$= e^{-\ln\frac{V}{K} - (r-q)\,(T-t)} = \frac{1}{e^{\ln\frac{V}{K} + (r-q)\,(T-t)}} = \frac{1}{\frac{V}{K} e^{(r-q)\,(T-t)}} = \frac{K}{V e^{(r-q)\,(T-t)}}$$

$$\frac{\partial C}{\partial V} = e^{-q(T-t)} N(d_1) + V\, e^{-q(T-t)} N'(d_1) \frac{\partial d_1}{\partial V} - K e^{-r(T-t)} N'(d_2) \frac{\partial d_2}{\partial V}$$

$$= e^{-q(T-t)} N(d_1) + V\, e^{-q(T-t)} N'(d_1) \frac{\partial d_1}{\partial V} - K e^{-r(T-t)} N'(d_2) \frac{\partial d_2}{\partial V}$$

$$= e^{-q(T-t)} N(d_1) + V\, e^{-q(T-t)} N'(d_1) \frac{1}{V\sigma\sqrt{T-t}} - K e^{-r(T-t)} N'(d_2) \frac{1}{V\sigma\sqrt{T-t}}$$

$$= e^{-q(T-t)} N(d_1) + V\, e^{-q(T-t)} N'(d_1) \frac{1}{V\sigma\sqrt{T-t}} - V \frac{e^{(r-q)\,(T-t)}}{e^{r\,(T-t)}} N'(d_1) \frac{1}{V\sigma\sqrt{T-t}}$$

$$= e^{-q(T-t)} N(d_1) + V\, e^{-q(T-t)} N'(d_1) \frac{1}{V\sigma\sqrt{T-t}} - V e^{-q(T-t)} N'(d_1) \frac{1}{V\sigma\sqrt{T-t}}$$

$$= e^{-q(T-t)} N(d_1)$$

3.2.10 American Call Options

In general, American call options are never worth less than an identical European counterpart because they offer more rights than European ones do. As already depicted in section “3.2.1 Definitions and Notation” the holder of an American call option is entitled to exercise not only on the maturity date but on every day up to the maturity date.

3.2.10.1 Non-Dividend Paying Underlying

According to Merton [MeRo73], American call options on a non-dividend paying stock are not exercised before the maturity date as they are more valuable alive than dead. This holds true because the option represents an insurance against a decrease in the price of the underlying. In other words, as long as there is no exercise the decrease does not hurt but afterwards this insurance gets lost. Moreover, the buyer of an option can invest the strike price in the market before exercising the option. If he/she is able to enlarge this period then he/she earns more money out of this investment [HuJo00]. As a result of Merton's proof, the models of the previous chapters can be applied for American calls too if the underlying pays no dividends:

Type of option	Author	Applicable on American Calls?
Option on a stock	Black and Scholes	Yes
Option on a stock which pays dividends	Merton	No
Option on a stock with an uncertain exercise price	Fischer	Only, if certain conditions are fulfilled
Exchange options	Margrabe	Yes
Compound options	Geske	Yes

Table 6: Applicability of valuation models for American call options
Source: Own Representation

The proof of the applicability of the Black-Scholes formula is given in [MeRo73], the restrictions of Fischer's model are listed in [FiSt78] and the validity of Margrabe's equation is depicted in [MaWi78]. As the valuation of compound options according to Geske consists of two options on non-dividend paying assets it can also be applied in the American case.

3.2.10.2 Dividend Paying Underlying

When dividends come into mind the situation changes and a premature exercise may be beneficiary. After an early exercise the former option holder and current shareholder receives the dividends to which he was not entitled before. Hence, an exercise before the expiry date makes only sense if the dividend rate is higher than the interest rate which could be earned on the exercise price. The literature provides some models for the valuation of American options on dividend paying assets which are listed in e.g. [KiMi95]. Roll [RoRi77], Geske [GeRo79b] and Whaley [WhRo81] developed a formula for an American call where the underlying share has exactly one known dividend payment. Moreover, Black [BlFi75] provides an approximation for the valuation of an American call on a share with n dividend payments. He makes use of the fact that an early exercise is only reasonable immediately before a dividend payment. So he values European call options for each pre-dividend date with maturity dates equal to these dates. For that it is only necessary to correct the stock price for the dividend payments. The value of the American call is then estimated by the maximum value of all European options [HuJo00]. This thesis uses for the valuation of American call options no closed form solution but the binomial tree method of Cox, Ross, and Rubinstein [CoRR79].

3.2.11 An Alternative: Binomial Option Pricing

In 1979 Cox, Ross, and Rubinstein presented a simplified approach for option pricing [CoRR79]. Contrary to the previously introduced continuous-time models they developed a model in discrete-time. This model assumes the following:

- A perfect financial market with no arbitrage opportunity
- The short-term interest rate is known and constant through time
  (the variable r refers to one plus the interest rate over a fixed length of time)
- The underlying stock price S follows a multiplicative binomial process over discrete periods (for details see 3.2.3.2 Stochastic Processes in Discrete Time). The return over each period is either u (with probability q) or d (with probability 1-q).

A graphical representation of such a stock price development looks as follows:

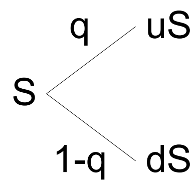


Figure 20: Stock price movement as a multiplicative binomial process
Source: [CoRR79]

Cox et al. split their study into two parts. First, they develop the basic model and second, extend it by taking dividend payments into consideration. This leads to the valuation of American options.

3.2.11.1 The Basic Model for European Call Options

The derivation of the model is explained best by starting with the simplest situation: the expiry date is only one period away. C refers to the current value of the call and depending on the stock price in one period, which is either uS or dS, and the exercise price K, the call's value in one period equals either C_u or C_d:

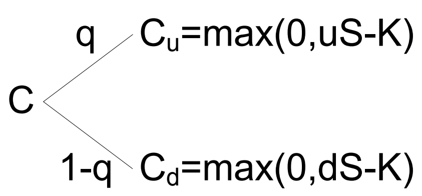


Figure 21: Call price movement
Source: [CoRR79]

As a next step Cox, Ross, and Rubinstein form a portfolio which is similar to the portfolio of Black and Scholes. This replicating portfolio has the same future returns as the call and contains Δ shares of stock as well as the dollar amount B in riskless bonds. The value of this replicating portfolio develops as follows over one period of time:

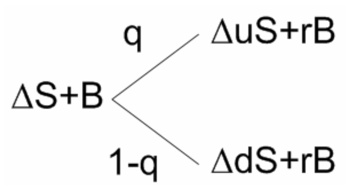


Figure 22: Replicating portfolio price movement
Source: [CoRR79]

As the portfolio is a perfect replication of the call

$$\Delta uS + r_f B = C_u \text{ and } \Delta dS + r_f B = C_d$$

must hold true at the end of period one. Using these equations the values for Δ and B are

$$\Delta = \frac{C_u - C_d}{(u-d)S} \text{ and } B = \frac{uC_d - dC_u}{(u-d)r_f}.$$

Choosing the two parameters in exactly this way results in the so called hedging portfolio. With this knowledge it is easy to calculate C, the value of the call option:

$$\begin{aligned} C &= \Delta S + B \\ &= \frac{C_u - C_d}{(u-d)} + \frac{uC_d - dC_u}{(u-d)r_f} \\ &= \left(\frac{r_f - d}{u-d} C_u + \frac{u - r_f}{u-d} C_d \right) / r_f \end{aligned}$$

Further simplification leads to

$$C = \frac{pC_u + (1-p)C_d}{r_f}$$

$$p = \frac{r_f - d}{u - d}$$

$$1 - p = \frac{u - r_f}{u - d}$$

As a result, the actual probability of the stock price movement (q) does not appear in the formula. Instead of it the risk-neutral probability (p) refers to the value which q would have in equilibrium if investors were risk-neutral. Hence, discounting is carried out with the risk-free rate of interest (r_f). Similar to the results of Black and Scholes the only random variable on which the call value depends is the stock price. Henceforth, the call's value does not depend on the investors' attitudes toward risk.

If the call option under consideration has two periods remaining before its maturity date then the situation changes as follows:

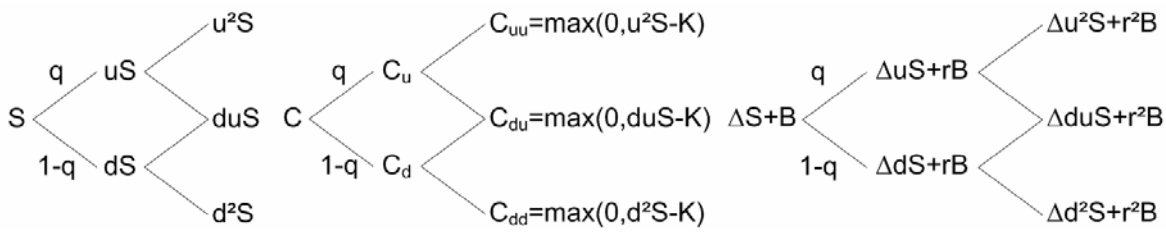


Figure 23: Price movements for two periods
Source: [CoRR79]

Using the formulas from above it is possible to go back through the tree and calculate first C_u and C_d. Afterwards it is possible to determine C:

$$C_u = \frac{pC_{uu} + (1-p)C_{ud}}{r_f},\ C_d = \frac{pC_{du} + (1-p)C_{dd}}{r_f} \text{ and } C = \frac{pC_u + (1-p)C_d}{r_f}$$

Here, it is necessary to adjust the replicating portfolio in every period, i.e. to recalculate the values for Δ and B.

An enlargement of the depicted procedure to n periods is possible without any modifications. However, if the number of periods grows very high the handling of the tree becomes quite uncomfortable. For a calculation of options with many periods before the expiry date Cox, Ross and Rubinstein transferred the recursive procedure into a formula using the binomial distribution function f(k; n, p) (see also 3.2.3.1 Fundamentals of Probability Theory):

$$C = S\, f(k; n, p') - K\, r^{-n}\, f(k; n, p)$$

$$f(k; n, p) = \frac{n!}{k!\,(n-k)!} p^{k} (1-p)^{n-k}$$

$$p = \frac{r-d}{u-d} \text{ and } p' = \frac{u}{r} p$$

a = the smallest non-negative integer greater than $\log(K/Sd^{n})/\log(u/d)$

if a > n then C = 0

This thesis applies only the manual procedure founded on two issues: first, the number of periods is not too high and second it is interesting to know which path of the tree will be gone in order to find the optimal exercise strategy (see the next session 3.2.11.2 Extension: Dividend Paying Underlying).

The next step is to extend the results from above by a consideration of very small time-intervals between the stock price changes. For that h represents the elapsed time between successive stock price changes. Moreover, n is the number of periods of length h prior to expiration:

$$h = \frac{\tau}{n}$$

When h becomes smaller, i.e. $n \to \infty$ the parameter r has to be adjusted to r^{h}. Since r changes, also p does which is depended on r. Moreover, the limiting case enables Cox, Ross, and Rubinstein to develop the following formulas for u and d:

$$u = e^{\sigma\sqrt{h}} \text{ and } d = e^{-\sigma\sqrt{h}}$$

These values presume a symmetric stock price movement, i.e. the stock price in period two after an up and a down movement (S * u * d) equals the stock price after a down and an up movement (S * d * u).

Another result of the limiting case is the proof that the multiplicative binomial model for stock prices includes the lognormal distribution. Furthermore, the binomial formula converges to the Black-Scholes formula.

3.2.11.2 Extension: Dividend Paying Underlying and American Options

The basic binomial model assumes that the underlying stock pays no dividends at all. In order to loosen this restriction δ denotes a constant dividend yield which the stock maintains on each ex-dividend date. Thus, the stock price is reduced by the amount $(S^*\delta)^v$ where v equals 1 if the end of the period is an ex-dividend date and v = 0 otherwise:

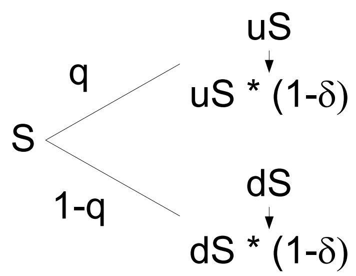


Figure 24: Stock price movement including dividends
Source: Own representation

Consequently, the value of the call slightly changes:

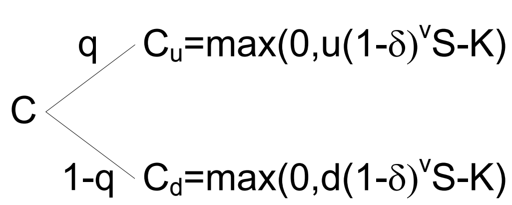


Figure 25: Call price movement including dividends
Source: [CoRR79]

As the formulas for u, d, p, and Δ are not affected, the valuation of an option on a dividend-paying underlying causes no new problems. Things become more interesting if the option is an American one. As dividends exist, premature exercise may be beneficiary. A small adaptation in the binomial model offers an easy way to calculate American options. In the following three-period-example a dividend payment occurs in period three. Hence, early exercise is possibly optimal in period two in order to prevent from the loss of value caused by the dividend in period three. Previously, C_u was defined as “let the option alive” and expressed as a function f (C_{uu}, C_{du}). Now the American option offers two possible actions and thus C_u is defined as “max (let the option alive; premature exercise)”:

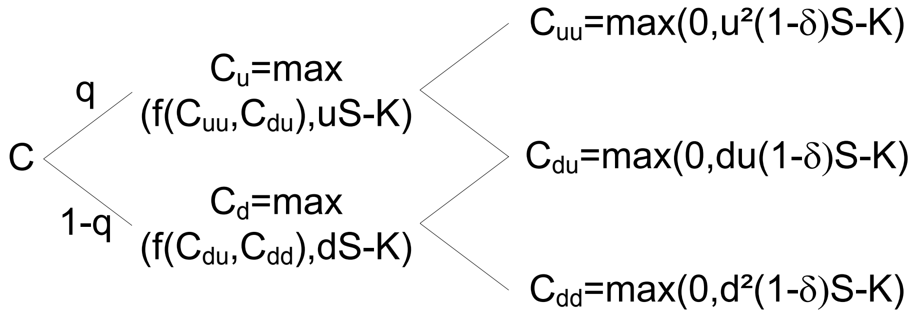


Figure 26: Call price movement including dividends for American options
Source: Own representation

3.2.11.3 Extension: Exchange Options

In 1991 Rubinstein published another extension of the binomial model [RuMa91]. Based on the study of Margrabe (see section 3.2.8) he developed an application for exchange options. The exchange option entitles its holder to give up an asset worth U_T at time T and receive in return an asset V_T. Hence, an exchange option represents a call option on V_T with exercise price U_T:

$$C(V_t, U_t, t) = \max (V_T - U_T, 0)$$

Analogously to Margrabe the price of asset V is defined in terms of U. This leads to a simple call option with exercise price equal to one:

$$\begin{aligned}\text{value of the option } &= C(V_t, U_t, t)\\ &= U_t * C(V_t / U_t, U_t / U_t, t)\\ &= U_t * \max (V_T / U_T - U_T / U_T, 0)\\ &= U_t * \max (V_T / U_T - 1, 0)\end{aligned}$$

Thus, a binomial tree can represent the value of the underlying asset as well as the option as follows:

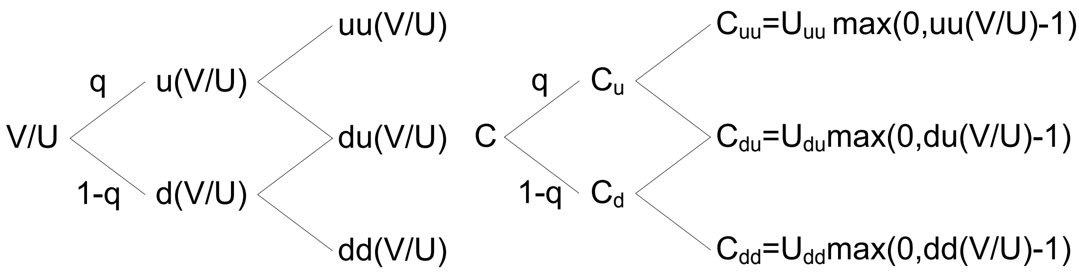


Figure 27: Exchange options – price movement for underlying and option value
Source: adapted from [RuMa91]

According to Margrabe, Rubinstein builds the hedging portfolio out of Δ_U units of U and Δ_V units of V and chooses the weights in a way to set the portfolio's return to zero. This leads to

$$C = U\,\delta_U\,\Delta_U + V\,\delta_V\,\Delta_V$$

where δ_U (δ_V) refers to one plus the payout rate of asset U (V) over each binomial period. The next figure shows the resulting binomial tree for an option over three periods with a dividend payment in the last period:

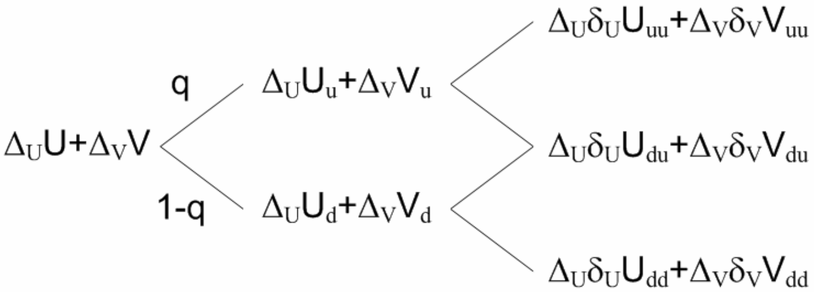


Figure 28: Price movement of the hedging portfolio for exchange options
Source: adapted from [RuMa91]

Analogously to the standard binomial model this hedging portfolio equals the option's value and it is possible to define:

$$\begin{aligned}
C_{uu} &= \Delta_U \delta_U U_{uu} + \Delta_V \delta_V V_{uu} \\
U_{uu}\max(0;uu(V/U)-1) &= U_{uu}(\Delta_U\delta_U + \Delta_V\delta_V uu(V/U)) \\
\max(0;uu(V/U)-1) &= \Delta_U\delta_U + \Delta_V\delta_V uu(V/U) \\
X_{uu} &= \Delta_U\delta_U + \Delta_V\delta_V uu(V/U) \\
&\text{and} \\
X_{du} &= \Delta_U\delta_U + \Delta_V\delta_V du(V/U)
\end{aligned}$$

Solving these simultaneous equations for Δ_U and Δ_V leads to:

$$\Delta_U = \frac{uX_{du} - dX_{uu}}{\delta_U (u-d)}$$
$$\Delta_V = \frac{X_{uu} - X_{du}}{\delta_V (u-d) u(V/U)}$$

After the determination of Δ_U and Δ_V and considering the possibility of an early exercise for American options one can calculate C_U as max (let the option alive; premature exercise):

$$\begin{aligned}
C_u &= \max\,(\Delta_U U_U + \Delta_V V_U ; V_U - U_U) \\
&= U_u \max\left(\frac{uX_{du} - dX_{uu}}{\delta_U (u-d)} + \frac{X_{uu} - X_{du}}{\delta_V (u-d) u(V/U)} u(V/U); u(V/U)\text{ - }1\right) \\
&= U_u \max\left(\frac{uX_{du} - dX_{uu}}{\delta_U (u-d)} + \frac{X_{uu} - X_{du}}{\delta_V (u-d)}; u(V/U)\text{ - }1\right) \\
&= U_u \max\left(X_{uu}\frac{\delta_U - d\delta_V}{\delta_V \delta_U (u-d)} + X_{du}\frac{u\delta_V - \delta_U}{\delta_V \delta_U (u-d)}; u(V/U)\text{ - }1\right) \\
&= U_u \max\left(\left(X_{uu}\frac{\frac{\delta_U}{\delta_V} - d}{(u-d)} + X_{du}\frac{u - \frac{\delta_U}{\delta_V}}{(u-d)}\right)\frac{1}{\delta_U}; u(V/U)\text{ - }1\right)
\end{aligned}$$

As a result C_u as well as C_d can be calculated and finally based on these results C is obtained:

$$\begin{aligned}
C_u &= U_u \max\left(\frac{pX_{uu} + (1-p)X_{du}}{\delta_U}; u(V/U)\text{ - }1\right) \\
C_d &= U_d \max\left(\frac{pX_{du} + (1-p)X_{dd}}{\delta_U}; d(V/U)\text{ - }1\right) \\
X_{uu} &= \max(0, uu(V/U) - 1] \\
X_{du} &= \max(0, du(V/U) - 1] \\
X_{dd} &= \max(0, dd(V/U) - 1]
\end{aligned}$$

$$p = \frac{\frac{\delta_U}{\delta_V} - d}{(u-d)}$$

$$C = U \max\left(\frac{pX_u + (1-p)X_d}{\delta_U} ; (V/U) - 1 \right)$$
$$X_u = \max(0, u(V/U) - 1]$$
$$X_d = \max(0, d(V/U) - 1]$$

The determination of the upward factor of the ratio V/U occurs in the usual way, i.e. $u = e^{\sigma\sqrt{h}}$. The necessary volatility equals $\sigma^2 = \sigma_V^2 - 2 \cdot \sigma_V \cdot \sigma_U \cdot \rho_{UV} + \sigma_U^2$ (for details see 3.2.8.3).

3.3 Valuation of Real Options

3.3.1 Overview about the Real Options Method

In general, the real options approach refers to the extension of the financial options theory to options on real or non-financial assets [AmKu99]. Myers [MySt77] used the term “real options” the first time when regarding corporate assets, particularly growth opportunities as call options. From today’s perspective his study is not considered to be part of mainstream real options research but it represents the starting point of a whole area of research in finance [BeHo04]. One of the first comprehensive works on contingent claim analysis was provided in 1994 by Dixit and Pindyck [DiPi94]. In his first edition of “Real options”, in 1996, Trigeorgis [TrLe00] offered a very important contribution which targets especially the practical implementation of a real options analysis (ROA). He points out that managerial flexibility is similar to a financial option as the managers can adapt their future actions in response to future developments. Similar to financial options the goal is to expand the investment’s value by improving its upside potential while limiting downside losses. Hence, contrary to the traditional static or passive NPV the ROA accounts for operating flexibility and strategic interactions. In this context Amram and Kulatilaka [AmKu99] point out that uncertainty is the manager’s friend as an increased uncertainty can lead to a higher investment value if the managers identify and use their options in order to response to unfolding events. In another work [AmKH99] they indicate that a ROA suits well when analysing investment decisions in IT as there are many options inherent. In general, literature distinguishes seven main categories of real options:

Category	**Description**
Option to defer	Management holds a lease on (or an option to buy) valuable land or resources. It can wait x years to see if output prices justify constructing a building, a plant, or developing a field.

Category	Description
Time-to-build option (staged investment)	Staging investments as a series of outlays create the option to abandon the enterprise in midstream if new information is unfavourable. Each stage can be viewed as an option on the value of subsequent stages and valued as a compound option.
Option to alter operating scale (e.g. to expand, to contract, to shut down and restart)	If market conditions are more favourable than expected, the firm can expand the scale of production or accelerate resource utilization. Conversely, if conditions are less favourable than expected, it can reduce the scale of operations. In extreme cases, production may be halted and restarted.
Option to abandon	If market conditions decline severely, management can abandon current operations permanently and realize the resale value of capital equipment and other assets on second-hand markets.
Option to switch	If prices or demand change, management can change the output mix of the facility (product flexibility). Alternatively, the same outputs can be produced using different types of inputs (process flexibility).
Growth options	An early investment (e.g. R&D, lease on undeveloped land or oil reserves, strategic acquisition, information network) is a prerequisite or a link in a chain of interrelated projects, opening up future growth opportunities (e.g. new product or process, oil reserves, access to new market, strengthening of core capabilities). Like inter-project compound options.
Multiple interacting options	Real-life projects often involve a collection of various options. Upward-potential-enhancing and downward-protection options are present in combination. Their combined value may differ from the sum of their separate values; i.e., they interact. They may also interact with financial flexibility options.

Table 7: Categories of real options
Source: [TrLe00, p. 2 – 3]

These seven types of real options emerge in various industries (see e.g. [MiPa02] p. 113) and thus there exists a huge number of different options in real life. Trigeorgis provides a more detailed description of the various options in his book [TrLe00] and a very good summary in one of his articles [TrLe05]. All real options have the set of parameters which influence their value in common. In comparison to the parameters of financial options, their equivalents for real options are:

Parameter of financial options	Equivalent for real options
Stock price (S)	The value of the underlying real asset. As it is usually not traded its value has to be computed using the DCF analysis.
Exercise price (K)	The investment which is necessary for the underlying real asset.
Risk free interest rate (r_f)	Same as for financial option
Time to expiration (τ)	Same as for financial option
Volatility of stock price (σ)	The variance of the returns of the underlying asset is not easy observable and requires special estimation techniques.
Dividend yield (q)	The convenience yield refers to a leakage in the value of the underlying asset arising from cash flows between decision points which the holder of the option cannot receive. Only the owner of the underlying asset obtains these cash flows.

Table 8: Parameters of financial options and their equivalents for real options
Source: Own representation based on [AmKu99], [BrMy03], [CoWS05], and [TrLe00]

Whereas the risk free interest rate and the time to expiration are the same for real options, the most important issue is to determine the value of the underlying asset. In most cases it is not possible to find a twin security on the market which has the same payoffs as the real asset. Here, the market asset disclaimer (MAD) approach offers help and uses a DCF analysis to calculate the value of the underlying [CoAn02]. In detail, it is necessary to calculate the present value (t=0) of the cash flows of the asset. Analogously, the option's exercise price is determined by discounting the cash flows of the investment to the maturity date of the option. The estimation of the volatility is quite difficult and is explained later. Finally, the convenience yield for one year represents the % value of the asset which is lost to the option holder as he/she is not entitled to certain cash flows. The determination of each parameter is demonstrated in detail in the cases studies which are available in "Part 5: Empirical Analysis". After the identification of all input parameters the valuation techniques of financial options (see 3.2 Valuation of Financial Options) are applicable to real options.

Although this study focuses on the investment decision it is worth to mention that the real options approach has its strengths also afterwards, i.e. in the active management of the existing options. Generally, it is possible to summarise the real options process as follows:

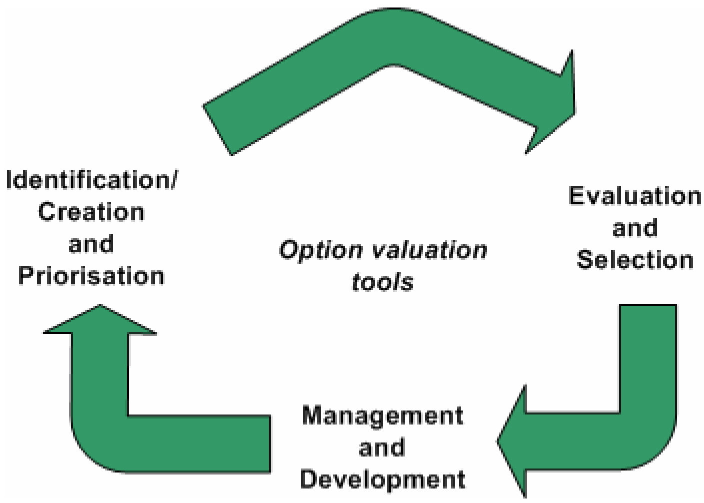


Figure 29: Real options process
Source: [RoTh04, p. 25]

An option can either exist and must be discovered or can actively be created by the management. As there might be many options it is necessary to concentrate the activities on the most important ones which are then evaluated. As a result of the calculation the decision-maker chooses whether or not e.g. to invest in an IIS. After this decision the next step is to manage the options which are created by the IIS. This management aims to improve the benefits by actively influencing those aspects which generate the value. Moreover, it monitors the options and decides when it makes sense to exercise them. As already depicted the current research deals with the steps one and two and thus explains their practical implementation in the context of an IIS valuation in chapter. More details about step three are available e.g. in the following literature: [HiYv06], [HoPr99], [LuTi98b], [MuJo06], [RoTh04], or [PrGu00].

3.3.2 Real Options Models

In order to account not only for the static or passive value of an investment Trigeorgis defines the expanded or strategic NPV [TrLe00]:

$$\text{Expanded (Strategic) NPV} = \text{Passive NPV} + \text{Option Premium}$$

Whereas the passive NPV contains the directly measurable expected cash flows the option premium considers the value of operating and strategic options stemming from active management, interaction effects of competition, synergy, and inter-project dependence. Hence, the real options approach does not replace a traditional NPV analysis but adds an important extension. For the calculation of the option premium the most prominent methods are:

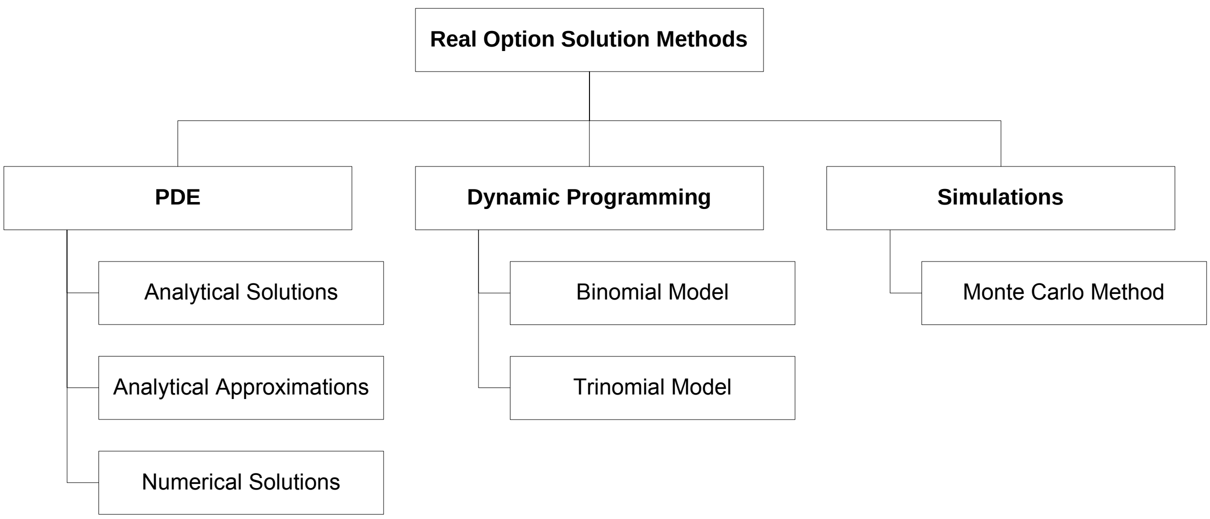


Figure 30: Real option solution methods
Source: Extended from [AmKu99]

The models of Black and Scholes, Merton, Fischer, Margrabe, and Geske (see 3.2) belong to the category of analytical solutions. They provide a closed form solution of a PDE which equates the change in the option value with the change in the replicating portfolio. If it is possible to formulate the problem as a PDE and to provide a closed form solution of the PDE then these methods represent the fastest way to calculate the value of the option. In some cases a modified partial differential equation is solved to obtain an analytical approximation. Furthermore, numerical solutions offer help if an analytical one is not possible by converting the PDE into a set of equations [AmKu99]. Examples of numerical solutions are the implicit and the explicit finite-difference methods. After translating the PDE into a set of equations these equations are solved iteratively. Hull [HuJo00], Trigeorgis [TrLe00], and Schulmerich [ScMa03] provide more details on these approaches. The binomial model, the trinomial model, and the Monte Carlo Method are also numerical solutions but are applicable in cases where it is not possible to write down a PDE at all. More information about the binomial model is presented in section 3.2.11 including the basic binomial model as well as extensions for dividends or convenience yields and exchange options. Hull [HuJo00] gives comprehensive information about trinomial trees where the stock price movement is not only either up or down but can also stay at its current level. Another way to approximate the underlying stochastic processes is the Monte Carlo simulation. A common course of action to value the option is as follows [HuJo00]:

1. Sample a random path for the underlying in a risk-neutral world
2. Calculate the payoff from the option
3. Repeat steps one and two to get many sample values
4. Calculate the mean of the sample payoffs to get an estimate of the expected payoff
5. Discount the expected payoff at the risk-free rate to get an estimate of the value of the option

For more details the interested reader is referred to the mentioned literature. The current thesis applies the methods of financial option pricing and thus concentrates on analytical solutions. A problem of these closed form solutions is that the decision-makers can hardly reconstruct the outcome, i.e. the number delivered by the formula. Hence, this research also uses binomial trees which are clearly more intuitive and easier to understand.

3.3.2.1 Extensions to Closed Form Solutions

For the completion of the toolkit for the option valuation two additional models are described in the following sections. The final toolkit of closed form solutions for European call options looks as follows (the new models are marked with “*”):

Type of option	Author	Reference
Ordinary option	Black and Scholes	[BlSc73]
Option with convenience yield	Merton	[MeRo73]
Option with uncertain exercise price	Fischer	[FiSt78]
Exchange option	Margrabe	[MaWi78]
** Exchange option with convenience yield*	*McDonald, Siegel*	*[McSi85]*
Compound option with or without convenience yield	Geske	[GeRo79a]
** Compound exchange option with or without convenience yield*	*Carr*	*[CaPe88]*

Table 9: Valuation models for European call options (extended)
Source: Own representation

3.3.2.1.1 Exchange Options with a Convenience Yield

In their study McDonald and Siegel [McSi85] develop an extension to Margrabe's formula [MaWi78] for the valuation of European exchange options. Their approach is the same as Margrabe's (see 3.2.8) but allows for the consideration of dividends or convenience yields of the underlying assets U and V. The value of the option which enables its holder to give up an asset U at time T and to receive in return an asset V is as follows (convenience yield of asset V and U: q_V and q_U):

$$C(V,U,t) = V\, e^{-q_V (T-t)}\, N(d_1)\;\; U\, e^{-q_U (T-t)}\, N(d_2)$$

$$d_1 = \frac{\ln\frac{V}{U} + \left(q_u - q_V + \frac{1}{2}\sigma^2\right)(T-t)}{\sigma\sqrt{T-t}} \quad \text{and}$$

$$d_2 = d_1 - \sigma\sqrt{T-t}.$$

$$\sigma^2 = \sigma_V^2 - 2\cdot\sigma_V\cdot\sigma_U\cdot\rho_{UV} + \sigma_U^2$$

Using this formula McDonald and Siegel investigate the investment problem of a company which has the option to shut down production if variable production costs exceed revenues.

3.3.2.1.2 Compound Exchange Option with and without Convenience Yields

Carr [CaPe88] combines the results of Margrabe [MaWi78] and Geske [GeRo79a] in order to create a valuation model for compound exchange options. In detail, the holder has the right to purchase a compound exchange option written on a simple exchange option. The simple exchange option consists again of the assets V and U and expires at T_2. The exercise price of the compound option equals I and its time to maturity is T_1 ($T_1 < T_2$). Moreover, the exercise price I follows the same stochastic process as U and is thus expressed as a portion x of U. The resulting formula is:

$$P = V / U$$

$$x = I / U$$

$$d_1(y,\tau) = \frac{\ln y + \frac{1}{2}\sigma^2(T-t)}{\sigma\sqrt{T-t}} \quad \text{and}$$

$$d_2(y,\tau) = d_1(y,\tau) - \sigma\sqrt{T-t}.$$

$$\sigma^2 = \sigma_V^2 - 2 \cdot \sigma_V \cdot \sigma_U \cdot \rho_{UV} + \sigma_U^2$$

$$\begin{aligned} C &= f(g(V,U), xU, t) \\ &= V\,B(d_1(P/\overline{P},\tau_1), d_1(P,\tau_2); \sqrt{\tau_1/\tau_2}) - U\,B(d_2(P/\overline{P},\tau_1), d_2(P,\tau_2); \sqrt{\tau_1/\tau_2}) - xU\,N(d_2(P,\tau_1)) \end{aligned}$$

Analogously to Geske's formula N() is the common cumulative standard normal distribution function, B(a, b, ρ) is a bivariate cumulative normal distribution function with upper integral limits a and b and correlation coefficient $\rho = \sqrt{\tau_1 / \tau_2}$. The critical value $\overline{P}$ is that value of P such that

$$V\,N(d_1(P, T - T^*)) - U\,N(d_2(P, T - T^*)) - xU = 0$$

$$P\,N(d_1(P, T - T^*)) - N(d_2(P, T - T^*)) - x = 0$$

For values of P less than $\overline{P}$ there will be no exercise of the option, while for values greater than $\overline{P}$ the holder will exercise the option. Again the calculation of the critical value is done with the Newton Raphson method using the following partial derivative of C:

$$\begin{aligned} \frac{\partial C}{\partial P} &= \frac{\partial\left(P\,N(d_1) - N(d_2) - x\right)}{\partial P} \\ &= N(d_1) + P\frac{\partial N(d_1)}{\partial P} - \frac{\partial N(d_2)}{\partial P} \\ &= N(d_1) + P\,N'(d_1)\frac{\partial d_1}{\partial P} - N'(d_2)\frac{\partial d_2}{\partial P} \end{aligned}$$

$$\frac{\partial d_1}{\partial P} = \frac{\partial \dfrac{\ln P + \frac{1}{2}\sigma^2(T-t)}{\sigma\sqrt{T-t}}}{\partial P} = \frac{\dfrac{\partial \ln P}{\partial P}\sigma\sqrt{T-t}}{\left(\sigma\sqrt{T-t}\right)^2} = \frac{\dfrac{1}{P}}{\sigma\sqrt{T-t}} = \frac{1}{P\,\sigma\sqrt{T-t}}$$

$$\frac{\partial d_2}{\partial P} = \frac{1}{P\,\sigma\sqrt{T-t}}$$

$$\frac{N'(d_1)}{N'(d_2)} = \frac{\frac{1}{\sqrt{2\pi}} e^{-0.5\, d_1^2}}{\frac{1}{\sqrt{2\pi}} e^{-0.5\, d_2^2}} = e^{0.5\, d_2^2 - 0.5\, d_1^2} = e^{0.5\,(d_1 - \sigma\sqrt{T-t})^2 - 0.5\, d_1^2}$$

$$= e^{-0.5 \cdot 2\, d_1 \sigma\sqrt{T-t} + 0.5\,\sigma^2 (T-t)}$$

$$= e^{-\left(\frac{\ln P + 0.5\,\sigma^2 (T-t)}{\sigma\sqrt{T-t}}\right)\sigma\sqrt{T-t} + 0.5\,\sigma^2(T-t)}$$

$$= e^{-\ln P} = \frac{1}{e^{\ln P}} = \frac{1}{P}$$

$$\frac{\partial C}{\partial P} = N(d_1) + P\, N'(d_1)\frac{\partial d_1}{\partial P} - N'(d_2)\frac{\partial d_2}{\partial P}$$

$$= N(d_1) + P\, N'(d_1)\frac{\partial d_1}{\partial P} - P\, N'(d_1)\frac{\partial d_1}{\partial P}$$

$$= N(d_1)$$

Moreover, Carr also develops an extension for dividends or convenience yields of the underlying assets U and V (convenience yield of assets U and V: q_U and q_V):

$$\begin{aligned} C &= f(g(V,U), xU, t) \\ &= V\, e^{-q_V \tau_2} B(d_1(Pe^{-q\tau_1}/\overline{P}, \tau_1), d_1(Pe^{-q\tau_2}, \tau_2); \sqrt{\tau_1/\tau_2}) \\ &\quad - U\, e^{-q_U \tau_2} B(d_2(Pe^{-q\tau_1}/\overline{P}, \tau_1), d_2(Pe^{-q\tau_2}, \tau_2); \sqrt{\tau_1/\tau_2}) \\ &\quad - xUe^{-q_U \tau_1}\, N(d_2(Pe^{-q\tau_1}, \tau_1)) \end{aligned}$$

The critical value $\overline{P}$ is that value of P such that

$$P\, e^{-q_V (T-T^*)} N(d_1(Pe^{-q(T-T^*)}, T-T^*)) - e^{-q_U (T-T^*)} N(d_2(Pe^{-q(T-T^*)}, T-T^*)) - x = 0$$

and the necessary derivative for the Newton Raphson method is given by:

$$\frac{\partial C}{\partial P}=\frac{\partial\left(P\,e^{-q_V(T-T^*)}N(d_1)-e^{-q_U(T-T^*)}N(d_2)-x\right)}{\partial P}$$

$$=e^{-q_V(T-T^*)}N(d_1)+Pe^{-q_V(T-T^*)}\frac{\partial N(d_1)}{\partial P}-e^{-q_U(T-T^*)}\frac{\partial N(d_2)}{\partial P}$$

$$=e^{-q_V(T-T^*)}N(d_1)+P\,e^{-q_V(T-T^*)}N'(d_1)\frac{\partial d_1}{\partial P}-e^{-q_U(T-T^*)}N'(d_2)\frac{\partial d_2}{\partial P}$$

$$\frac{\partial d_1}{\partial P}=\frac{\partial\dfrac{\ln P+(-q+\frac{1}{2}\sigma^2)(T-t)}{\sigma\sqrt{T-t}}}{\partial P}=\frac{\dfrac{\partial \ln P}{\partial P}\sigma\sqrt{T-t}}{\left(\sigma\sqrt{T-t}\right)^2}=\frac{\dfrac{1}{P}}{\sigma\sqrt{T-t}}=\frac{1}{P\,\sigma\sqrt{T-t}}$$

$$\frac{\partial d_2}{\partial P}=\frac{1}{P\,\sigma\sqrt{T-t}}$$

$$\frac{N'(d_1)}{N'(d_2)}=\frac{\dfrac{1}{\sqrt{2\pi}}e^{-0.5\,d_1^2}}{\dfrac{1}{\sqrt{2\pi}}e^{-0.5\,d_2^2}}=e^{0.5\,d_2^2-0.5\,d_1^2}=e^{0.5\,(d_1-\sigma\sqrt{T-t})^2-0.5\,d_1^2}$$

$$=e^{-0.5\cdot 2\,d_1\sigma\sqrt{T-t}+0.5\,\sigma^2(T-t)}$$

$$=e^{-\left(\frac{\ln P+(-q+0.5\,\sigma^2)(T-t)}{\sigma\sqrt{T-t}}\right)\sigma\sqrt{T-t}+0.5\,\sigma^2(T-t)}$$

$$=e^{-\ln P+q(T-t)}=\frac{e^{q(T-t)}}{e^{\ln P}}=\frac{e^{q(T-t)}}{P}$$

$$\frac{\partial C}{\partial P}=e^{-q_V(T-T^*)}N(d_1)+P\,e^{-q_V(T-T^*)}N'(d_1)\frac{\partial d_1}{\partial P}-e^{-q_U(T-T^*)}N'(d_2)\frac{\partial d_2}{\partial P}$$

$$=e^{-q_V(T-T^*)}N(d_1)+P\,e^{-q_V(T-T^*)}N'(d_1)\frac{\partial d_1}{\partial P}-P\,e^{-q(T-T^*)}\,e^{-q_U(T-T^*)}N'(d_1)\frac{\partial d_1}{\partial P}$$

$$=e^{-q_V(T-T^*)}N(d_1)+P\,e^{-q_V(T-T^*)}N'(d_1)\frac{\partial d_1}{\partial P}-P\,e^{-q_V(T-T^*)}N'(d_1)\frac{\partial d_1}{\partial P}$$

$$=e^{-q_V(T-T^*)}N(d_1)$$

Finally, Carr offers some possible areas of application for his formulas including a real investment decision.

3.3.2.2 Extensions to Binomial Trees

Whereas section 3.2.11.3 already offers a binomial model for exchange options with convenience yields there was so far no explanation concerning binomial trees for compound options or compound exchange options. Fortunately, this enhancement is much easier than those of the closed form solutions from above. In order to calculate the value of a compound option written on a simple option, the following steps are necessary regardless if the simple option is an exchange option or not:

1. Drawing of a tree showing the price movement of the underlying asset of the simple option
2. Backward calculation of the value of the simple option using a second tree
3. Consideration of an additional decision point in this second tree which represents the decision to exercise the compound option

In this course of action only step three is new and no other steps are necessary as the strike price of the compound option follows the same process as the one of the simple option. An application of this process is available in the case studies of section 5.4. Other good references concerning this topic are [GrRo06] or [CoAn02].

3.3.2.3 Volatility Estimation

Section 3.3.2 concludes with models for estimating the volatility of the option's underlying asset. As the volatility is one of the crucial input parameters its determination is a well-addressed topic in research. For financial options there generally exists enough historical data about the price of the stock for the calculation of the volatility. As this holds not true for real options other methods for the assessment are necessary. At this point it is very important to keep in mind that the volatility refers to the variance of the returns of the underlying asset, i.e. the benefits generated by the investment. Thus, the volatility is not the same as the volatility of the company's stock. Previous research mentions three important types of estimation techniques [MiPa04]:

- Twin security approach
- Modified scenario analysis
- Monte Carlo simulation

The **twin security approach** is applicable for investments where an appropriate twin security is available in the market. In such a case the historical return distribution of the security serves as a proxy for the real asset. This approach assumes that if the real asset had been publicly traded then it would have varied in a similar fashion as the twin security [MiPa04].

Beside the historic volatility it is also possible to use the implied volatility. For that all parameters for the option calculation (except the volatility) as well as the option's price are observed in the market. Afterwards the option valuation formula is solved for the volatility parameter [HePa02]. As it is not possible for every real investment to find a twin security with the same future cash flows this study will not apply this estimation technique. However, there may exist other types of real options where the price of a commodity is the only relevant source of uncertainty. In such cases market data can be used to estimate volatility [GoPe06].

In their study Miller and Park [MiPa04] present a **modified scenario approach** where they deploy the properties of the geometric Brownian motion to calculate the volatility. First, it is necessary to obtain two estimations (V^A and V^B) of the asset's value at time T, i.e. at the maturity date of the option. Here V_T is defined as the present value of all cash flows C, discounted at time T (k = discount rate, n = number of periods):

$$V_T = C_0 + PV_0 \, (1+k)^T$$
$$PV_0 = \sum_{t=1}^{n} \frac{C_t}{(1+k)^t}$$

According to the properties of the geometric Brownian motion V_T will be lognormal with parameters (μ, s^2). Consequently, it is possible to write:

$$P\,(V_T < V^A) = p_1 \text{ and } P\,(V_T < V^B) = p_2 \text{ with } V^A > V^B$$
$$P\,(\ln V_T < \ln V^A) = p_1 \text{ and } P\,(\ln V_T < \ln V^B) = p_2$$

The transformation into a standard normal variable

$$z = \frac{\ln V_T - \mu}{s}$$

yields

$$P\,(z < z^A) = p_1 \text{ and } P\,(z < z^B) = p_2.$$

Moreover, the properties of the geometric Brownian motion ($s^2 = \sigma^2 T$) result in

$$z^A = \frac{\ln V^A - \mu}{\sigma\sqrt{T}} \text{ and } z^B = \frac{\ln V^B - \mu}{\sigma\sqrt{T}}.$$

These two equations lead to the following definition of σ:

$$\sigma = \frac{\ln \frac{V^A}{V^B}}{(z^A - z^B)\sqrt{T}}$$

If V^A represents the best case scenario, which means that the true value will be less than V^A in 9 of 10 cases (p = 90%), then $z^A = 1.28$. Contrary, if V^B represents the worst case scenario, i.e. the true value will be more than V^B in 9 of 10 cases (p = 10%), then $z^B = -1.28$. A disadvantage of this approach is its dependency on T. If T rises e.g. for one period, then both V^A and V^B increase by the discount rate. Hence, the term V^A / V^B does not change and the outcome for σ decreases. As a result the higher the option's maturity the lower the volatility and thus the lower the option value. This conflicts with the basic principle of options that a later maturity date leads to an increased option value. In order to overcome this problem this study follows Copeland's and Antikarov's [CoAn02] argumentation. They recommend performing the estimation on the basis of the return between period 0 and 1:

$$PV_1 + C_1 = PV_0 \, e^r$$
$$X_1 = PV_0 \, e^r$$

Defining X^A (X^B) as the best (worst) case scenario for X_1 yields to ($z^A = 1.28$ and $z^B = -1.28$):

$$X_1 \in \left[X^A, X^B\right]$$
$$X^A = PV_0 \, e^{r+1.28\sigma} \text{ and } X^B = PV_0 \, e^{r-1.28\sigma}$$

Solving the equations for X^A and X^B leads to the following results:

$$\sigma = \frac{\ln \frac{X^A}{PV_0} - r}{z^A}, \sigma = \frac{\ln \frac{X^B}{PV_0} - r}{z^B}$$

For T=1, $C_0 = 0$, and PV_0 being the median (z-score = 0) this is a special case of Miller's and Park's result:

$$\sigma = \frac{\ln \frac{V^A}{V^B}}{(z^A - z^B)\sqrt{T}} = \frac{\ln \frac{X^A + C_0 e^r}{(PV_0 + C_0)e^r}}{z^A - 0} = \frac{\ln \frac{X^A}{PV_0 e^r}}{z^A} = \frac{\ln \frac{X^A}{PV_0} - r}{z^A}$$

Moreover, Copeland and Antikarov use a **Monte Carlo simulation** which predicts the return between period 0 and 1:

$$r = \ln \frac{PV_1 + C_1}{PV_0} = \ln \frac{PV_0(1+k)}{PV_0}$$

PV_0 is obtained by an ordinary NPV calculation which represents the "normal case". During the simulation the denominator (PV_0) is constant and the numerator (PV_1+C_1) varies according to the simulation parameters. Copeland and Antikarov recommend 1000 runs and after their completion e.g. the mean as well as the desired variance of r, are available. A detailed description of the steps to be performed during the simulation is given in [CoAn02]. The following remarks present only a rough guideline. The basic idea is that the uncertainty of the asset's value is represented by the probability distribution of the PV. The PV consists of various cash flow streams which are caused by different input factors. As these input factors are uncertain it is the uncertainty of each of the factors that determines the uncertainty of the whole asset. In order to capture this relationship the Monte Carlo simulation comprises of the following steps:

1. Creation of a model for the calculation of the asset's PV for the normal case
2. Definition of the simulation parameters:
    a. Definition of the assumptions for each input factor,
       i.e. the factors' distribution including the mean and the variance

b. Definition of the forecast,
   i.e. the return between period 0 and 1
3. Execution of the simulation

The calculation of the PV in step 1 does not need further instructions. This PV model for the normal case contains all factors and their means. The variance of the input factor I is again calculated based on the best/worst scenario estimates I^A / I^B:

$$I_T = I_0 e^{rT}$$
$$I_T \in \left[I^A, I^B \right]$$
$$I^A = I_0 \ e^{rT+1.28\sigma\sqrt{T}} \text{ and } I^B = I_0 \ e^{rT-1.28\sigma\sqrt{T}}$$

$$\sigma = \frac{\ln\frac{I^A}{I_0} - rT}{z^A\sqrt{T}}, \sigma = \frac{\ln\frac{I^B}{I_0} - rT}{z^B\sqrt{T}}$$

After the definition of the forecast-variable it is possible to start the simulation which provides the desired result, namely the variance of the asset's return. Herat and Park [HePa02] also propose a Monte Carlo simulation for the estimation of the volatility. They use the same formula for r but handle also the denominator (PV_0) in addition to the numerator (PV_1+C_1) as random. As both random variables are independent it is necessary to use different random number sequences. Moreover, they suggest 5000 runs in order to ensure a correct result.

Godinho [GoPe06] points out that both methods suffer from some limitations. First, each approach assumes a constant volatility. If the volatility changes with time the method is easy to adapt but if it changes with the investment value then serious problems arise. However, the option valuations of this research presuppose a constant volatility like most other researchers do. Second, it is only possible to calculate and simulate r if PV_1 and PV_0 do not change signs. Third, Godinho explains a significant upward bias in the results of Copeland and Antikarov as well as Herat and Park. The bias is due to the use of the ex post cash flows instead of their expected values for the calculation of PV_1. The ex post or future cash flows of the periods two to n are unknown at year one. Thus, instead of them the information available at year one, i.e. the expected values of the cash flows, should enter the calculation of PV_1:

$$PV_1 = \sum_{t=2}^{n} \frac{E_1(C_t)}{(1+k)^t}$$

Only this way leads to a measure for the year-one value and not to an ex post value for a specific scenario. As PV_0 is not constant in the approach of Herat and Park, the upward bias also occurs there and their method suffers from a larger error than the method of Copeland and Antikarov. Godinho holds PV_0 constant but also uses here the expected cash flows:

$$PV_0 = \sum_{t=1}^{n} \frac{E_0(C_t)}{(1+k)^t}$$

If there is no analytical expression available for $E_0(C_t)$ it is possible to use a simulation to compute PV_0. In order to calculate PV_1 Godinho presents two different algorithms. In each of them the main goal is to simulate the investment's behaviour in the first year and, for each iteration, to estimate the expected future cash flows $E_1(C_t)$ according to the information available at the end of the first year. The first algorithm represents a two-level simulation and looks as follows [GoPe06, p. 20]:

```
numiter1 ← number of iterations of the first level simulation
numiter2 ← number of iterations of the second level simulation
For i = 1 to numiter1
        Simulate the investment behaviour in the first year
        For j = 1 to numiter2
                Simulate the investment behaviour after the first year, until the end
        Next j
        Calculate the average cash flows after the first year
        Use the average cash flows after the first year to calculate PW1
        Use PW1 to calculate a sample of k1
Next i
Calculate the volatility as the standard deviation of k1
```

The major shortcoming of this approach is the length of computer time that is required: 50,000 iterations for the first level and 50,000 iterations for the second one lead to a total of $50{,}000^2 = 25 * 10^8$ iterations. As this may be impracticable the second algorithm considers two single-level simulations each consisting of 50,000 trials [GoPe06, p. 22]:

```
numiter1 ← number of iterations of the first simulation
numiter2 ← number of iterations of the second simulation
For i = 1 to numiter1
```

```
        Simulate the investment behaviour for all the years
Next i
From the results of the first simulation, estimate a model (for example, using linear regression)
    that calculates the conditional expectation of PW1 given year-1 information
For i = 1 to numiter2
        Simulate the investment behaviour in the first year
        Use the estimated model to calculate the expected value of PW1
        Use the expected value of PW1 to calculate a sample of k1
Next i
Calculate the volatility as the standard deviation of k1
```

The present research applies the second algorithm instead of the first one and uses a linear regression model to calculate PW1. According to Godinho, this leads to results which are precise enough. However, more sophisticated methods may sometimes provide better outcomes.

Furthermore, Mun [MuJo06] describes five different estimation techniques. His logarithmic cash flow return or logarithmic stock price return approach equals the twin security approach requiring a lot of cash flow data and is thus usually applied for financial options only. The logarithmic present value returns approach is the same as Copeland's Monte Carlo simulation. An additional procedure is the GARCH (Generalized Autoregressive Conditional Heteroskedasticity) model. It is mainly used for tradable assets and requires a lot of data. In a nutshell this technique considers the variance of the current error term to be a function of the variances of the previous time period's error terms [Wiki07a]. Another way of estimating the volatility is Mun's management assumptions and guesses approach. For T = 1 and median = mean this one is again the same as the modified scenario analysis:

$$z = \frac{\text{Percentile Value - Mean}}{\sigma}$$

$$\sigma = \frac{\text{PercentileValue} - \text{Mean}}{z}$$

$$= \frac{r^A - r}{z^A} = \frac{\ln \frac{V^A}{V_0} - r}{z^A}$$

The fifth estimation technique of Mun is the market proxy approach which relies on market data and is very straightforward and thus not applied in this research.

To summarise, this study uses Godinho's method as the leading one for estimating the volatility. In addition, it also applies the modified scenario approach and the simulations according to Copeland and Antikarov as well as Herat and Park. These three techniques are easier to deploy but have to be tested in terms of their accuracy.

3.3.3 Application of ROA on IIS

Applying the real options concept to integrative information systems fits very well as the benefits arise mainly not from the IIS but from the follow-on investments which use the platform. This situation refers to a growth option where the IIS generates call options on follow-on opportunities which are realised if it is profitable. Thus, investing in an IIS is analogous to buying an option on those follow-on applications [DaKM00]:

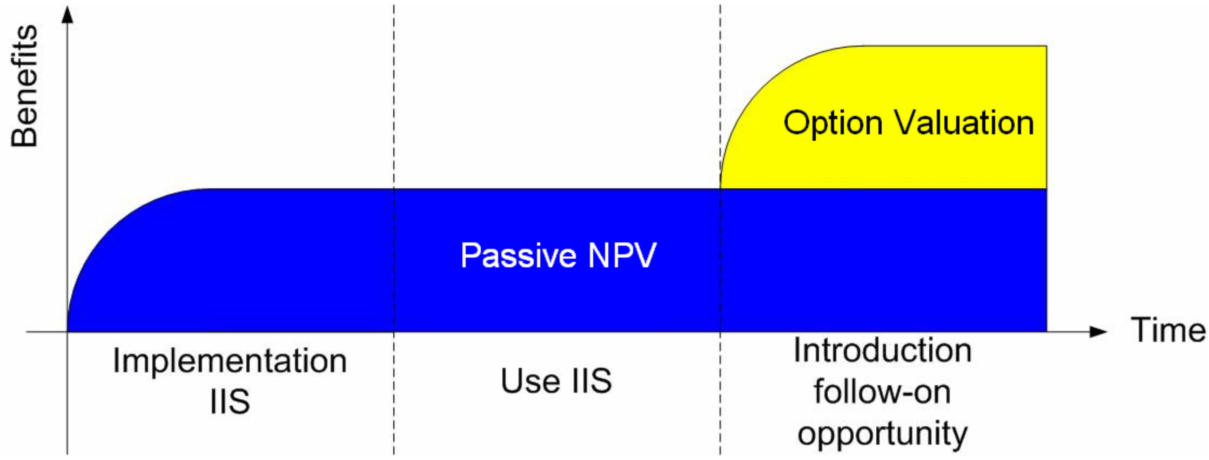


Figure 31: IIS as growth option
Source: Adapted from [TaAl98]

At the time of implementation of the IIS the follow-on investments can be highly unknown, i.e. either they will generate high benefits or produce losses. Such situations with a notable degree of uncertainty are ideal for an ROA. In contrast to a NPV analysis the decision maker is not forced to regard the follow-on investment as certain. Instead, the ROA accounts for his possibility to invest only in cases where it is favourable by deferring the actual implementation decision to a later point in time (= the maturity date of the option). Hence, the passive NPV of the IIS is not burdened by negative follow-on values (no exercise of the option) but supplemented by positive follow-on values (exercise of the option). Fichmann [FiRo04] presents in his study 12 factors which positively influence the value of growth options by increasing the volatility and the managerial flexibility. He groups them into four perspectives on organisational innovation, namely technology strategy perspective (e.g. radicalness or strategic importance of affected products), organizational learning perspective (e.g. knowledge barriers or learning-related endowments), bandwagon perspective (e.g. prospects for network dominance of the technology), and adaption perspective (e.g. flexibility or divisibility). Beside growth options, managerial flexibility can also generate other options like staging the necessary investments or deferring them.

As a result, the investment decision is based on the expanded NPV which is defined similar to Taudes [TaFM00] as follows:

Expanded (Strategic) NPV = Passive NPV of the IIS
+ Growth option value of the follow-on opportunities
+ Option value of additional managerial flexibility

These option values can be calculated using the previously introduced closed form solutions or binomial tree approximations. On the one hand closed form solutions are very comfortable as they provide one formula which delivers the result directly. On the other hand, their outcome is hard to understand and reproduce. This is a big disadvantage in cases where the ROA concept is new in a company and decision-makers must be convinced about its results. Here, the binomial tree approximation shows its strengths by generating trees which are easy to understand. However, the important point is that a ROA helps to justify infrastructure investments. As Trigeorgis [TrLe00] points out this is even more important due to the increased decentralisation of decision making and partitioning into separate divisions. Such a structure comes along with decentralised resource allocation often favouring a piecemeal approach at the cost of infrastructure investments. The individual divisions see only the immediate tangible costs that affect their own budget but do not always recognize the remote, intangible, or contingent benefit derived from such investments, especially when these are spread across the entire organisation.

Detailed questions about the actual application of the valuation methods like the precision of the binomial tree approximation, or when to use which formula (uncertain costs yes/no, convenience yields yes/no, compound options yes/no) will be investigated in the following sections of this thesis (see “Part 4: Development of a Research Framework” and “Part 5: Empirical Analysis”).

3.3.4 Applicability and Criticism

Beside the clear advantage of accounting for managerial flexibility the most critical points of ROA are the underlying assumptions that stem from the financial world (see 3.2.5). The three most unsafe ones are:

- the underlying asset is traded in a perfect financial market with no arbitrage opportunity
- the underlying asset price follows a geometric Brownian motion
- the volatility of the underlying asset is constant over time

Whereas these restrictions are clear for financial options they cause some problems in the world of real options. The financial option valuation models use the risk-free interest rate for discounting the expected cash flows. This is possible as the underlying asset is traded in a perfect financial market with no arbitrage opportunities. Contrary, **the underlying asset of real options is usually not traded** and thus the question arises whether or not it is possible to use financial valuation methods in a ROA. On the basis of the argumentation of Mason and Merton [MaMe85] Benaroch and Kauffman [BeKa99] indicate, that the goal of capital budgeting is to determine what the project's cash flows would be worth if they were traded regardless of whether a project is actually traded. Hence, the goal is to measure the contribution of the cash flows to the firm's market value and the shareholders' wealth. Biases in the investment values resulting from wrong calculations would lead to arbitrage opportunities that the market will correct even when the project is not traded. This correction might take a form like described in the following two examples: in the first example the results are too low (e.g. because the analyst uses a too high cost of capital) and the company does not invest and fails to exploit potential returns. The firm will then trade for less than it is worth and eventually there will be an economic agent who purchases the firm. In the second example, the results are too high and the company invests by receiving too little money. If this occurs regularly within the firm it will fail in the marketplace.

Moreover, Trigeorgis [TrLe00] reveals that a ROA imposes no new but the same assumptions as a NPV analysis in this area. For the determination of the discount rate a twin security with the same risk characteristics is identified and the NPV analysis uses its equilibrium required expected rate of return which is usually estimated by the CAPM. Furthermore, given the prices of the project's twin security, management can, in principle, replicate the returns to a real option by building a portfolio of the twin security and borrowing money at the riskless rate. The equilibrium value of an option on a non-traded project must be the no-arbitrage value of the option on its traded twin security. The only restriction is that in the risk-averse world of the real asset it is possible that the asset's actual growth rate α falls below its equilibrium growth rate α^*. The resulting shortfall is analogous to a constant dividend yield $\delta = \alpha^* - \alpha$. Hence, Trigeorgis concludes that any contingent claim on an asset, whether traded or not, can be priced in a world with systematic risk by replacing its actual growth rate, α, with a certainty-equivalent rate by simply subtracting a risk premium RP from α. As a result discounting with the risk-free rate is still possible but instead of the actual expectation (the asset's value with growth rate α) the risk-neutral expectation (the asset's value with growth rate α – RP) is subject to discounting. To summarise, this risk-neutral growth rate $\hat{\alpha}$ is expressed as $\hat{\alpha} = \alpha - RP = r - \delta$. According to Benaroch and Kauffman [BeKa00], it is very difficult to estimate such a shortfall. Using a sensitivity analysis they show that the influence of δ on the option value is very small. Hence, current research in the field of real options usually abstains from corrections of the actual growth rate. Additionally, Taudes [TaAl98] argues that it is not necessary to be overly precise as the value of the underlying asset itself is only an approximation and this estimate is the really critical issue. However, Hubalek and Schachermayer [HuSc01] investigate the general case when an underlying X is not a traded asset but its price is highly correlated with that of a traded asset Y. As simply replacing X by Y is not the optimal hedging strategy they develop alternative strategies which minimize the variance of the hedging error. The resulting model is very complex and they do not offer a comprehensive application on a real world case.

The second assumption of closed form solutions is that **the underlying asset of the option follows a lognormal distribution**. Otherwise the Black-Scholes pricing produces biases, i.e. the results are either too high or too low, which quantification requires an exact modelling of the asset's distribution [BeKa99]. Lander and Pinches [LaPi98] mention existing studies using other stochastic processes than a geometric Brownian motion for the underlying asset. Typical ones are mean reverting stochastic processes or jump stochastic processes. It remains unclear when to use which one and how severe the process influences the option value. Moreover, they point out that not only the process selection but also the estimation of the parameters required is critical. However, Marathe and Ryan [MaRy05] check whether a particular time series data follows a geometric Brownian motion or not. They conclude that for some data sets the GBM process may be appropriate (e.g. electric utility data, passenger data) but for others not (e.g. cell phone revenue data, internet host data).

Third, **the volatility of the underlying asset might not be constant** in the real world. The most likely case is that the volatility decreases over time when the time to maturity is significant large relative to the end of the useful life of the option's underlying. As uncertain events become known to the analyst option pricing becomes less attractive [BeKa99]. However, as Benaroch and Kauffman [BeKa99] point out the binomial model enables the analyst to model parameters such as the stochastic process and the volatility by programming more complex behaviours into the binomial tree. Another interesting issue concerning the relationship between volatility and option value is presented by Kumar [KuRa96]. In most cases it holds true that an increase in the volatility leads to a higher option value. Given an follow-on opportunity which equals an option to exchange the risky asset C (Costs) against the risky asset B (Benefits) the option value can increase but also decrease the higher σ_B (σ_C) depending on the sign of the term $\sigma_B - \rho_{BC}\ \sigma_C$ ($\sigma_C - \rho_{BC}\ \sigma_B$). Hence, Kumar's conclusion is that it is not always attractive to select riskier second-stage projects. Additionally, Willershausen et al. [WiMS07] examine the influence of the volatility in detail and demonstrate that a higher volatility does not need to result in a higher option value. Mac Millan et al. [MPMT06] address the fact that not only an increase in the benefits' volatility (σ_B) but also in the costs' volatility (σ_C) raises the option value. According to them a high volatility of the costs should penalise the option calculation. Hence, in cases where σ_C is higher than σ_B they correct σ_B as follows: $\sigma_{B\ corrected} = \sigma_B / \sigma_C * \sigma_B$. Unfortunately, they do not explain why the correction is made this way. Moreover, one can argue that a higher value of σ_C also allows for lower costs and thus for a higher option value. However, the case studies of this research show values of σ_C which are clearly below σ_B.

Financial option valuation models assume only one source of uncertainty, that is the volatility. As this might not be enough for real options, some researchers account in their models for more than one source of uncertainty but unfortunately, this complicates the valuation a lot. Copeland and Antikarov [CoAn05] provide an interesting contribution in this field by postulating that it is possible to bundle the different uncertainties in one factor, namely the volatility. Their argumentation starts with Samuelson's proof that suitably anticipated prices fluctuate randomly: in a market with complete information the current stock price is the result of expectations about future cash flows. Discrepancies of these expectations are subject to random events and thus also the variance of the expected returns is subject to random events. Then they recommend bundling several sources of uncertainty in a single one which is the asset's return. If this is possible and if this return is subject to random events then an analyst can use a binomial model for a ROA. However, as Cobb and Charnes [CoCh04] point out it can already be quite difficult to consolidate the uncertainties because of possible correlations among the input factors.

Another possible difference between financial and real options might be the exclusiveness of ownership. Taking a financial option its holder has not to worry about competition for the underlying asset, i.e. he/she has an exclusive right to exercise. This holds not true for all real options like in circumstances where the holder shares his option with competitors. An example of such a situation would be the introduction of a new product to penetrate a new geographic market [TrLe00]. However, in case of implementing an IIS and its follow-on opportunities the assumption of an exclusive ownership seems appropriate. That eases the calculation a lot because otherwise a more complicated (e.g. game-theoretic) analysis would be required.

De Jong et al. [DeRV99] propose a very negative point of view concerning ROA by mentioning three main problems. First, the estimations of the volatility and the NPV of the follow-on investment are difficult. So the problems with NPV analyses are not solved but it is still necessary to predict cash flows and to determine an appropriate discount rate. Second, the formula is too simplistic to have "real life" value. Examples therefore are assumptions like a constant interest rate or the non-existence of transaction costs. Third, many users label the model as complex and unintuitive. This creates problems for managers who have to understand the nature of their results. However, they also point out that it is important to think of options like not implementing a follow-on opportunity if things go wrong.

All in all ROA is clearly not impeccable but a step in the right direction and Tallon et al [TKLWZ02, p. 138] summarise the situation as follows: *"Despite any initial misgivings, the benefits of ROA remain attractive to IT managers who are repeatedly faced with difficult investment decisions involving technical and organizational uncertainty, multiple forms of risk and incomplete information. Any technique that allows IT decision makers to consider risk and uncertainty factors in their decisions is a positive step. What remains unresolved, however, is the extent to which ROA represents an appropriate and useful tool for decision* *makers seeking to make more informed investment decisions."* Hence, the benefit of a ROA is not only represented by the number produced but also by the insights gained e.g. about follow-on opportunities. Fichmann et al. [FiKT05] support this view and emphasise the importance of options-thinking by listing several guidelines for implementing it in project management.

Part 4: DEVELOPMENT OF A RESEARCH FRAMEWORK

4.1 Previous Research and Research Assumptions

4.1.1 Previous Research

Baecker and Hommel [BeHo04] provide an excellent review of the existing literature in the area of real options. They assess more than 350 studies addressing the topic of ROA in various ways. This large number indicates that there exist various forms of real options which are spread across different industries. Hence, the current study focuses on previous research in the area of information technology and especially on growth options stemming from IIS. The pioneering work in this field was written in 1991 by **Dos Santos [DoBr91]**. He introduced the first real options model for the valuation of IT-investments which benefits arise merely through follow-on opportunities. He argues that an investment in a new technology is negative but enables one or more follow-on opportunities which show a positive value and thus justify the initial investment. For a valuation of this option he proposes Margrabe's formula (see 3.2.8) for an European option to exchange one asset, the implementation costs C, against another one, the benefits B of the follow-on opportunity [DoBr91]:

$$\text{Option value} = B_1N(d_1) - C_1N(d_2)$$

B_1.......current value of the expected benefits of the follow-on project
C_1 ......current value of the expected development costs of the follow-on project
N() .....cumulative standard probability density function

The study proposes no clear approach of how to estimate the volatility but mentions only the possibility of a subjective estimate done by the potential users. Similarly, there is no obvious suggestion for the assessment of the correlation between B and C, i.e. the degree to which the revenues from the second-stage project are dependent on the development costs of the project. Furthermore, Dos Santos provides no application for a real world case. **Panayi and Trigeorgis [PaTr98]** investigated a real world case and applied a ROA. Unfortunately they do not explain in detail which model the use for the valuation. The object of consideration is the development of an information system which supports the telecommunications development plan of the telecommunications authority of Cyprus. This system generates a growth option, namely a further expansion of the telecommunications network. The NPV of the initial investment is negative and also the NPV of the follow-on investment. Applying a ROA leads to a positive option value of the network expansion and even to a positive strategic

NVP. The first application of a closed form solution on a real world case was presented in 1999 by **Benaroch and Kauffman [BeKa99]**. In their study they investigate an option to defer a bank's investment in the POS debit market. Using the results of several option calculations with the Black-Scholes formula for different maturity dates they approximate the value of this American option. The necessary volatility estimation is again obtained by an estimation of the management. In their following research Benaroch and Kauffman investigated the same case but added sensitivity analyses to check for the robustness of their results [BeKa00].

Based on the initial research of Taudes [TaAl98] **Taudes, Feurstein, and Mild [TaFM00]** apply the Black-Scholes formula to investigate an investment in an IIS, namely SAP/R3. This platform enables four follow-on investments which transform the negative passive NPV of the IIS to a positive strategic NPV. The estimate for the volatility is again quite straightforward using a managerial assessment. Another IIS investment was examined by **Dai, Kauffman, and March [DaKM00]** using the binomial model. Their case study is about Northern States Bank's decision to implement an object-oriented middleware, the "customer relationship system tool", in order to integrate the existing heterogeneous systems. After one year four possible follow-on investments emerge which are not valued separately but in one NPV analysis. Their overall NPV is positive but not high enough to justify the initial investment. Hence Dai et al. also perform an option valuation which leads then to a slightly positive strategic NPV.

A guideline of applying a ROA to information systems is available in the study of **Li and Johnson [LiJo02]**. They investigate the investment opportunity of an electronic brokerage firm of building a wireless trading infrastructure. The company has the option to defer the investment but other competitors also have the possibility to enter the new market. In this situation, i.e. shared opportunity with high IT switching costs, the authors recommend to consider the strategic growth options and to adopt a "wait-and-see" strategy. When the technical uncertainty reduces and thus the switching costs decline they advise an early pre-emptive investment, i.e. to exercise the option. The valuation itself is quite straightforward by using a binomial model to determine four European option values with different maturities.

Schwartz and Zozaya-Gorostiza [ScZo03] also do not concentrate on investments in IIS but differentiate in general between IT acquisition projects and IT development projects. The former one refers to cases where the necessary investment occurs once or during a short period of time in comparison to the overall life of the technology. Contrary, the latter term comprises situations where the company keeps investing for a longer period of time. The authors develop valuation models for both cases but point out that most investments in IT infrastructure can be evaluated using the first model. Similar to Dos Santos [DoBr91] they account not only for uncertain benefits but also for uncertain costs. The resulting model for IT acquisition projects is quite complicated as they not only make adjustments using a risk-premium but also regard not the PV of all cash flows as underlying asset but only the PV of all future cash flows. Furthermore, they consider an American option and as a result it is not possible to provide a closed form solution. Moreover, for an application it is necessary to calculate additional input parameters like the positive or negative growth rate of the costs and benefits. Finally, their work concludes with an application of the developed model on the case of Benaroch and Kauffman [BeKa99].

Influenced by the work of Kulatilaka and Perotti [KuPe98] **Clemons and Gu [ClGu03]** developed a framework for the valuation of technology investments that enable the future deployment of a strategy in case that conditions arise which make this strategy desirable. In order to capture these conditions their model accounts for the behaviour of customers and game-theoretic factors like the actions of competitors. Avoiding the restrictions of closed form solutions the approach provides a heuristic solution and thus stays as general as possible. Moreover, they do not impose arbitrary restrictions but establish an economic model for their general framework. This allows to provide arbitrary complex sets of conditions as inputs and thus for simulations to evaluate ex ante values of contingent investment strategies given these conditions. The application of the framework is shown for the credit card industry where an attacking bank launches a pricing strategy and a creative bank has various strategic responses. Each strategy is measured in comparison to a third bank which ignores the changes and maintains the status quo.

An additional ROA of information systems is provided by **Miller, Choi, and Park [MiCP04]**. They do not value an investment in an IIS but an investment of a Korean company in the Korean information superhighway infrastructure. Similar to IIS this infrastructure generates growth options, namely two expansion investments at t = 2 and t = 4. The NPV of phase one and two is clearly negative whereas the NPV of phase three is slightly positive. However, the overall NPV is negative and thus this analysis would suggest not undertaking the investment. For the valuation of the European compound option the authors use Geske's formula and for the estimation of the volatility parameter they follow Copeland's and Antikarov's [CoAn02] Monte-Carlo simulation model. The resulting option value for phases two and three is then high enough to justify the initial investment.

In 2005 **Chen et al. [CSGW05]** offered a further contribution to research on ROA by investigating a digital government investment of the state Utah. The base investment in the infrastructure is made only for supportive parties and enables an expansion to others in the future. In order to value these growth options they develop a model which is similar to the Black-Scholes formula but considers estimation errors in the value of the underlying at t = 0 and t = option maturity. The results obtained from the option calculation justify the infrastructure investment which has a slightly negative NPV.

A case study that investigates the values of options to extend a general contractor's software platform is given by **Ekström and Björnsson [EkBj05]**. By pointing out that the architect's adoption rate represents the major risk and that this uncertainty is external they use a binomial tree model in combination with a simulation. The ERP system under consideration enables the company to invest in several follow-on opportunities. For a closer examination the authors choose an investment in a request for information (RFI) application. The static NPV of this application is negative but an option valuation leads to a clearly positive option value.

Dai, Kauffman, and March [DaKM07] present a very new study in the area of valuing IT infrastructure investments. They investigate especially investments in middleware technologies and the growth options associated with them. The real options model is different to the previous ones as they do not consider the cost-benefit relationship only but also the market factors influencing these costs and benefits. The investments in the infrastructure and the follow-on opportunity aim to provide a certain product or service which demand is subject to market conditions. The value of the infrastructure is the difference between providing this product or service with the infrastructure and its follow-on investment or without them. As it is necessary for the option valuation to estimate difficult parameters especially for the market conditions an application on a real world case would be very interesting. Unfortunately, such an application is not available in their work.

Another very new application of ROA in the area of IT is available in the work of **Harmatzis and Tanguturi [HaTa07]**. They examine company's A deferral option and company's B expansion option in the wireless industry using a slightly modified Black-Scholes formula. A's option to defer the expansion of 2.5G to 3G wireless networks does not produce a positive strategic NPV. The option of company B to expand its 2.5G network using alternative technologies like WLAN's is analysed for two scenarios. Whereas the first scenario results in a negative strategic NPV the second scenario shows a positive one.

Although not investigating investments in IT the following studies also provide helpful information. In an early contribution Luehrman [LuTi98a] provides a very clear application of the Black-Scholes formula on a hypothetical investment. Johnstone [JoDa02] gives a good example of using a binomial tree model to value an exchange option including a convenience yield in the area of public sector outsourcing. An analysis of a compound growth option with a binomial tree model in the R&D sector is available in [HePa02] including a detailed volatility estimation. Miller and Park [MiPa04] investigate a delay, a growth option, and a compound option of a company in the aerospace maintenance, repair, and overhaul industry. Beside an in depth binomial tree approximation they also illustrate their way of assessing the volatility in detail. Another elaborate implementation of the binomial tree model including an extensive determination of the volatility is available in [CoAn05]. Grabner and Rothwell [GrRo06] use also a binomial tree to assess a licensing and construction option on a nuclear plant. They show how a binomial tree can be used to examine a compound exchange option with a convenience yield. In a recent contribution of the year 2007 Armada et al. [ArKP07] value an American exchange option using a closed form solution which is based on Carr's formulas [CaPe88 and CaPe95]. They compare three different projects which aim to install a retail business facility and contain an option to defer the investment necessary.

To summarise, the following table illustrates the most important works, their results, and their valuation parameters. If available then the studies show a negative NPV of the initial investment but a positive strategic NPV resulting from the option analysis. It is observable that the follow-on opportunity is undervalued when using a NPV analysis instead of a ROA. Concerning the model there appear closed form solutions as well as binomial trees either determining an European or an American option. In case of uncertain costs the assessment of the correlation coefficient is always done quite straightforward by subjective estimates. This is also the most common way for appraising the volatility.

Author	Reference	Description	NPV initial investment	NPV follow-on	Strategic NPV	Volatility	Uncertain costs	Correlation coefficient	Convenience yield	Option Type	Model
Dos Santos	[DoBr91]	New information technology creates follow-on opportunities				subjective estimate	yes	subjective estimate	no	European	Closed form (Margrabe)
Panayi, Trigeorgis	[PaTr98]	New information system creates growth option	< 0	< option value	> 0	subjective estimate	no		no	European	Not clear
Benaroch, Kauffmann	[BeKa99]	Option to defer investment for entrance in POS debit market				subjective estimate	no		yes	American	Closed form (approximation of the American value using n Black-Scholes results)
Taudes, Feurstein, Mild	[TaFM00]	IIS generates follow-on opportunities	< 0	< option value	> 0	subjective estimate	no		no	European	Closed form (Black-Scholes)
Dai, Kauffman, March	[DaKM00]	IIS generates follow-on opportunities	< 0	< option value	> 0	subjective estimate	no		no	European	Binomial model
Li, Johnson	[LiJo02]	Opportunity to defer the investment in an IT infrastructure			> 0	subjective estimate	no			American	Binomial model (approximation of the American value using n European options)
Schwartz, Zozaya-Gorostiza	[ScZo03]	Option to defer investment for entrance in POS debit market				subjective estimate	yes	subjective estimate	yes	American	Modification of Dos Santos
Clemons, Gu	[ClGu04]	IT infrastructure generates strategic options									Market Competition
Miller, Choi, Park	[MiCP04]	IT infrastructure generates two follow-on opportunities	< 0	< option value	> 0	Monte Carlo [CoAn02]	no		no	European	Closed form (Geske)
Chen, Sheng, Goreham, Watanabe	[CSGW05]	IT infrastructure generates one follow-on opportunities	< 0		> 0	subjective estimate	no		no	European	Closed form (Extension of Black-Scholes)
Ekström, Björnsson	[EkBj05]	IIS generates follow-on opportunities		< option value		Market data	no		no	European	Binomial model (and simulation)
Dai, Kauffman, March	[DaKM07]	IIS generates follow-on opportunities									Market Competition
Harmatzis, Tanguturi	[HaTa07]	New technology generates option to expand			scenario 1: <0 scenario 2: >0	Market data	no		yes	European	Closed form (Extension of Black-Scholes)

Table 10: Summary of previous research
Source: Own representation

4.1.2 Research Assumptions

This section develops assumptions which are subjects of investigation in the case studies. On the one hand they reflect expectations which originate from the literature review of the prior section. On the other hand it is their aim to investigate the possible extent of simplification of a real options analysis of an IIS-investment.

Previous research points out that most of the benefits of an IIS stems from its follow-on opportunities. As a result the NPV of an IIS alone is expected to be negative:

Assumption 1: Negative NPV of IIS
A NPV analysis of an investment in the IIS without a consideration of the follow-on opportunities leads to a negative NPV.

Moreover, the NPV method would undervalue the follow-on opportunity in comparison with an option analysis:

Assumption 2: NPV smaller than option value
The application of the NPV-method for the valuation of a follow-on opportunity leads to significantly undervalued results, i.e. there is at least a deviation of -10% from the value received by the real options analysis.

The existing studies for the valuation of real options use either a closed form solution or some sort of binomial tree. This suggests that the binomial tree approximation provides results which are precise enough.

Assumption 3: Binomial tree approximation sufficient
If a closed form solution is possible then a binomial tree (maximum of 20 nodes) approximates this result with sufficient precision, i.e. there is a maximum deviation of +/-5% from the value received by the closed form solution.

The subsequent two assumptions deal with the complexity of the option valuation model. There exist two possible sources of simplification: first the question arises whether or not it is sufficient to use a simple valuation model which does not account for convenience yields or dividends, respectively.

Assumption 4: Model complexity – no convenience yields
A proper determination of the input parameters eliminates the need to account for convenience yields (“dividends”) in the real options analysis, i.e. there is a maximum deviation of +/-5% between the simple non-dividend option model and the complex option model.

The second possible simplification concerns the treatment of the implementation costs of the follow-on opportunity. Here the decision maker has to choose between a complex model which incorporates for uncertain costs and a less complex model which treats the costs as fixed. Existing studies show the benefit in the option value by regarding the costs as uncertain. Thus, the following point assumes that it is worth accounting for uncertain costs.

Assumption 5: Model complexity – uncertain implementation costs
The implementation costs should not be regarded as fix, i.e. the option value calculated with fixed implementation costs deviates at least -5% from the option value determined with uncertain implementation costs.

Besides the difference in the results between a “simple model” (i.e. no convenience yields and fixed implementation costs) and a “complex model” (i.e. convenience yields and uncertain implementation costs) it is interesting to determine if they lead to different investment decisions:

Assumption 6: Equal investment decision – simple model vs. complex model
The investment decision received using the simple model is equal to that received by the complex model.

Moreover, at least the investment decision which results from a NPV analysis should be different compared to the decision relying on the option value:

Assumption 7: Different investment decision – NPV vs. option value
The investment decision received using the NPV is different from the decision received by the complex option valuation model.

Regardless of the option valuation model it is compulsory to estimate the volatility. Although there exist studies which use quite straightforward methods for the estimation it seems that these methods lead to significant errors compared to Godinho’s more elaborate Monte-Carlo estimation [GoPe06].

Assumption 8: Volatility estimation according to Godinho
It is necessary to use Godinho’s approach to estimate the volatility. The option value computed with other estimation techniques deviates at least +/-10% from the option value received using Godinho’s approach.

An additional input parameter which is difficult to estimate but obligatory for the valuation of options with an uncertain exercise price is the correlation coefficient. Contrary to the volatility previous research provides no special techniques for its determination and in most cases managers only provide a rough estimation.

Assumption 9: Rough estimation of the correlation coefficient
The correlation coefficient has a minor impact on the option value, i.e. a change of +/-30 percentage points leads to a modified option value of less than -/+5%. Thus, it is possible to determine its value by a rough estimation.

Another interesting issue is whether or not a deferral of the follow-on opportunity's implementation increases the option value substantially. As the IIS limits the useful life of the follow-on opportunity it is expected that the deferral causes no critical enhancement.

Assumption 10: Minor benefit of a postponed implementation of the follow-on opportunity
As the expected useful life of the follow-on opportunity is limited by the expected useful life of the IIS it is advisable to start with its implementation as soon as possible, i.e. the European option value with the earliest possible maturity date deviates from the American option value not more than +/-10%.

The next assumption deals with the question whether or not a small estimation error in the PV of the cash flows causes a higher shift in the option value than the use of a complex model instead of a simple model.

Assumption 11: Small estimation error of the PV has less influence than the model complexity
An estimation error of +/-5% in the PV of the benefits/costs results in a lower change in the option value compared to the difference obtained between the simple and the complex model.

4.2 Case Study Methodology

4.2.1 General

As depicted in the beginning of the document, this work uses the case study approach as research method. Thus, each of the assumptions from above will be inspected in the case studies of the next section. In order to ensure a solid proceeding it is necessary to define a process model for the real options analysis which enables the examination of each assumption as well as the ROA itself. The application of this process model is compulsory for every case study and the model does not contain any simplifications for the ROA of IIS. Instead, it allows for all combinations in the following areas of complexity:

1. Type of model: closed form approach vs. binomial tree
2. Model complexity 1: consideration of convenience yields vs. no consideration
3. Model complexity 2: uncertain implementation costs vs. fixed implementation costs
4. Volatility estimation: Monte Carlo methods vs. straight forward methods
5. Type of option: American option vs. European option

The results of the individual cases will show in which of these five areas it is possible to ease the ROA of IIS and thus the desired simplified process model can be developed. The next figure shows a summary of this course of action:

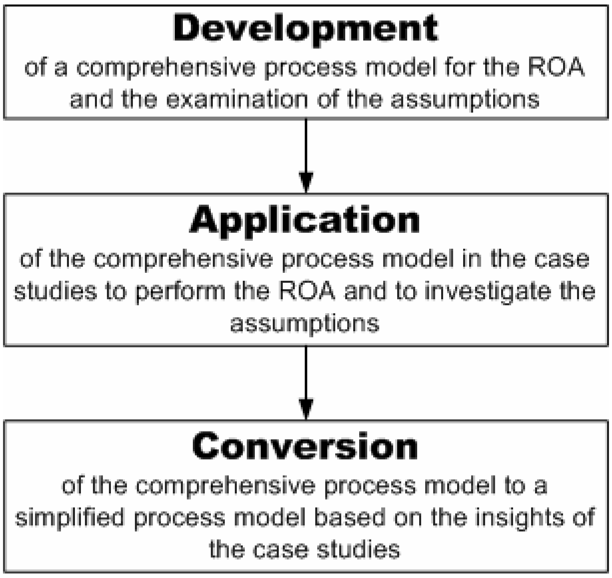


Figure 32: Case study methodology – Course of action
Source: Own representation

To recapitulate, the application of this process enables to reach the main goal of this research, namely to create a simplified model for the valuation of investments in IIS using the real options approach.

4.2.2 Comprehensive Process Model

Section “3.3 Valuation of Real Option” gives a description of the real options analysis as well as its application on IIS. Now the question arises how one can perform a ROA for a particular case. The need of a process model comes up which describes every step that is necessary for the analysis. The following figure presents the model that will be used by all case studies:

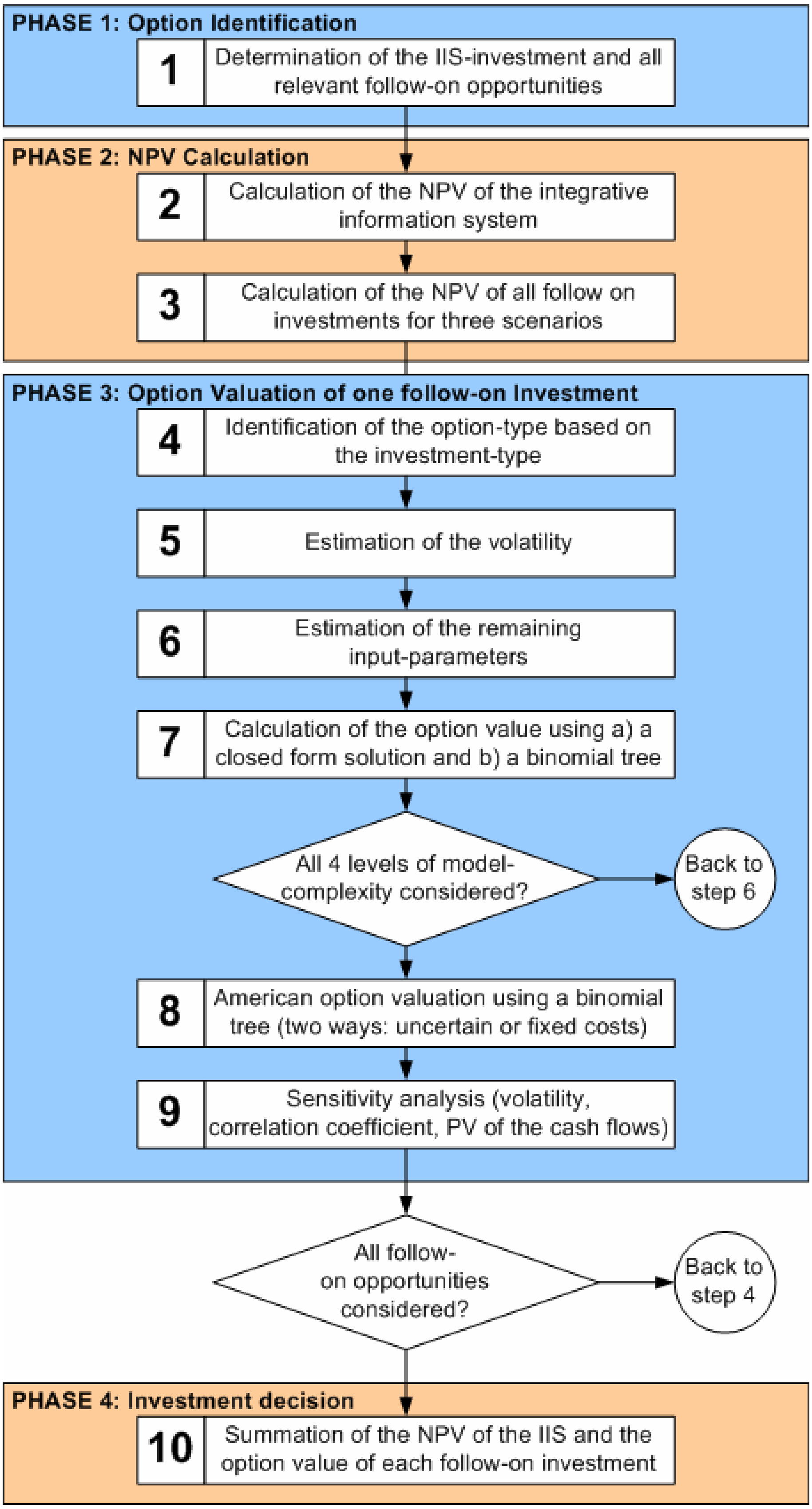


Figure 33: Comprehensive process model for the ROA
Source: Own representation

It contains four phases comprising ten steps which are assigned to the assumptions as follows:

#	Assumptions	Steps which are used for the examination
1	Negative NPV of IIS	Step 2
2	NPV smaller than option value	Comparison of the result of step 3 to the results of step 7
3	Binomial tree approximation sufficient	Comparison of the results of step 7
4	Model complexity – no convenience yields	Comparison of the various results of step 7 which are obtained by the repeated executions of steps 5 to 7 (see jump label “Back to step 6” in the process model)
5	Model complexity – uncertain implementation costs	Comparison of the various results of step 7 which are obtained by repeated executions of steps 5 to 7 (see jump label “Back to step 6” in the process model)
6	Equal investment decision – Simple model vs. complex model	Comparison of the decisions that would be made in step 10 based either on the simple model or on the complex model
7	Different investment decision – NPV vs. option value	Comparison of the results of step 2 and 3 to the decision of step 10
8	Volatility estimation according to Godinho	Comparison of the results of step 9 based on the different outcomes of step 5
9	Rough estimation of the correlation coefficient	Step 9
10	Minor benefit of a postponed implementation of the follow-on opportunity	Comparison of the results of step 7 and step 8
11	Small estimation error of the PV has less influence than the model complexity	Comparison of the results of step 9 and the differences in the results between the simple and the complex model

Table 11: Assignment of the assumptions to the steps of the process model
Source: Own representation

The following chapters describe the individual phases and steps of the process model in detail.

4.2.2.1 Phase 1: Option Identification

In order to value an option it is necessary to identify this option in a first step. Luehrman [LuTi98a] mentions that it takes some experience to recognize the options which are integral parts of conventional investments. As a basic guideline he proposes two ways to reveal the options. First, a look at the investment’s description might be of help to discover e.g. staged investments. Second, an examination of the investment’s cash flows can be useful to find discretionary capital expenditures which may have an optional character. Similarly to the first point, Amram and Kulatilaka [AmKu99] recommend to start with a description of the decisions including the triggers and the authorities executing the decisions. An interesting study in this area is provided by Benaroch [BeMi02] who recommends to find out the underlying risks first and afterwards to map these risks to options. However, Rokke [RoTh04] points out that it is also possible to actively create options, e.g. by splitting up and re-structuring decisions or by introducing additional future alternatives.

When identifying/creating options associated with an integrative information system the following rudimentary questions will be of help:

- Is the investment in the IIS split into more than one phase?
- Is there an expense during the IIS-investment which is optional but necessary to enable a follow-on opportunity?
- Which applications can be implemented in the future based on the IIS?
- Are the investments of these follow-on opportunities split into more than one phase?

If the decision maker discovers many options then he/she should focus on the most promising ones in order to reduce the overall effort. It is advisable to visualize the results of this phase using a figure which shows the existing options and when they are due. This helps not to lose the big picture and to consider the dependencies during the valuation correctly.

4.2.2.2 Phase 2: NPV Calculation

Phase two starts with the NPV analysis of the IIS and concludes with the NPV calculation of all follow-on investments from phase one. As a preparation for the estimation of the volatility in step five the NPV of the follow-on investments is determined for three scenarios: best, normal, and worst scenario. The cash flows of the normal scenario represent the median, i.e. 50% of the actual values are above and 50% of the values are below them. The best scenario uses values for the cash flows where only 10% of the actual ones are above and 90% of the actual ones are below them. Contrary, the worst scenario implies that 90% of the actual values are above and only 10% below the scenario's values. Section "3.1.1 Net Present Value" explains the NPV method in adequate detail.

4.2.2.3 Phase 3: Option Valuation

The goal of this phase is to calculate the option value of a particular follow-on opportunity. Hence, for n independent follow-on opportunities it is required to perform phase three n times. The previous sections mention different option valuation models and thus, the dilemma is to choose the correct one. First, the characteristics of the investment are examined in order to determine the type of option. This is important as the option type has a major influence on the selection of the valuation model. The following steps refer to the estimation of the input parameters and the calculation itself. They are repeated for four models which differ in their complexity. The first run uses fixed implementation costs and does not account for convenience yields (i.e. dividends). The second one values the option with uncertain implementation costs whereas the third one takes convenience yields into account. In the last run the calculation addresses both issues and thus represents the most complex valuation. Finally, phase 3 concludes with the calculation of the American option value. This enables an examination whether or not a variable start of implementation creates a benefit in comparison to a fixed one.

4.2.2.3.1 Step 4: Identification of the Option-Type

Regarding the design of the investments in the IIS and in the follow-on opportunity as well as the relationship between them it is possible to deduce different investment types. The simplest form contains one investment in the IIS and afterwards one possible investment in the follow-on opportunity. This kind ("Type A: Single/Single") generates the following cash flow structure:

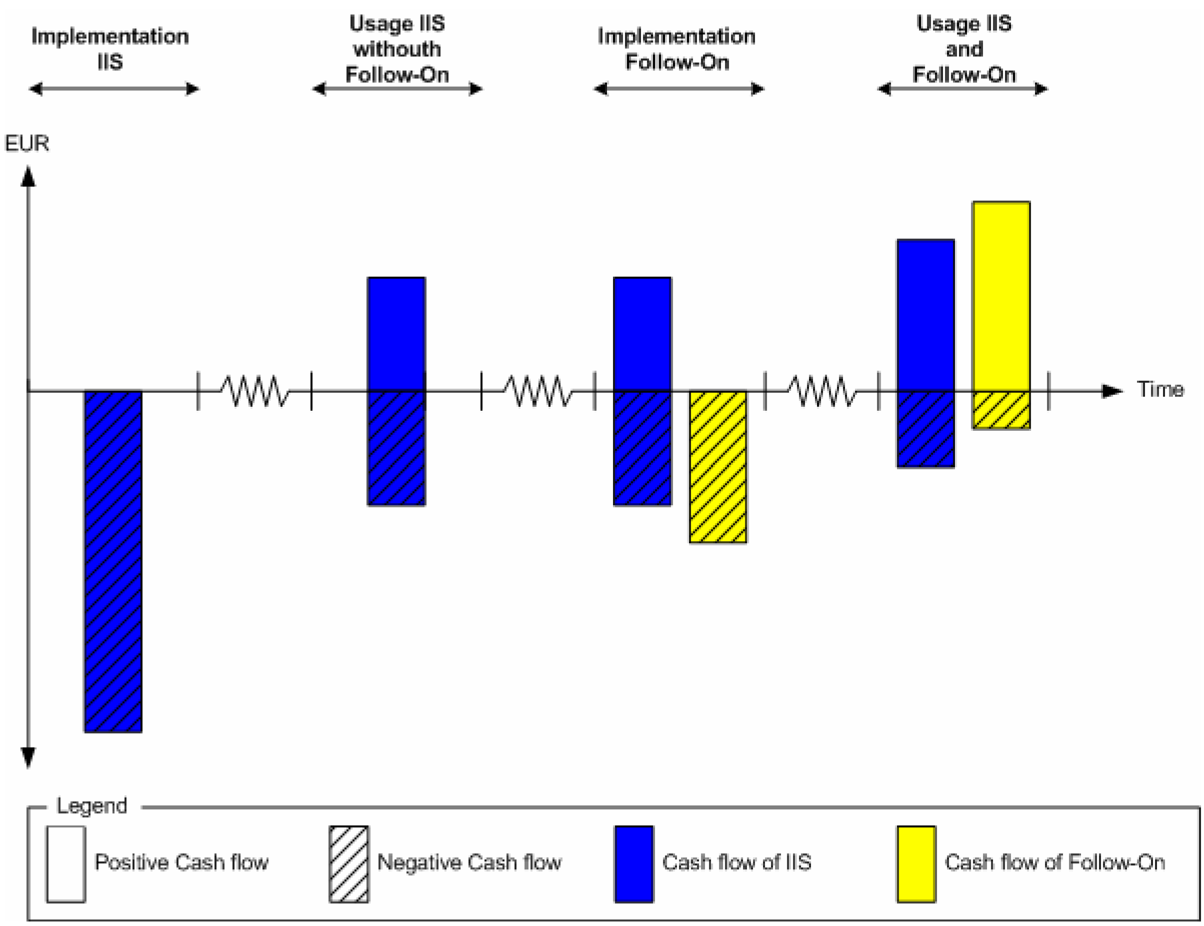


Figure 34: Investment type A (single/single) – cash flow structure
Source: Own representation

During the first stage the company has to make a capital-expenditure in order to build or buy the IIS. In the second stage the usage of the IIS begins and so the IIS starts to produce benefits, i.e. positive cash flows in addition to negative cash flows like operating costs. Next, there is the option to implement a follow-on opportunity in stage three. Finally, both systems are in use and create positive and negative cash flows until the end of their useful life. Stage four represents this state.

The second type ("Type B: Staged/Single") refers to the case where there is also one investment in the follow-on opportunity but two or more investments in the IIS. This is called a staged investment and it offers the possibility to continue the implementation by spending additional money after the completion of one stage. Another example of type B is described by Taudes [TaAl98] who mentions so called preparation costs in the IIS which enable the functionality of the follow-on opportunity. On the one hand this means that this investment creates the possibility for the follow-on opportunity but on the other hand if these costs are not spent then it is not possible to realise the follow-on opportunity at all. However, for the valuation it is advisable to reduce the number of stages to two because this eases the computation a lot. Moreover, in practice there usually will not be more than three or four stages. Thus, in addition to the first stage (the start of implementation) the largest one is chosen as second stage. Those stages which are not regarded separately are included in the first or second one respectively. In option thinking type B represents a compound option. The initial investment in the IIS creates the opportunity to implement a second stage which in turn offers the possibility to implement the follow-on opportunity.

Moreover, it comes another type ("Type C: Single/Staged") into mind when there is only one investment in the IIS but two or more follow-on investments. The staged investment in the follow-on opportunity also comprises situations where one follow-on investment is the prerequisite for another follow-on investment. If there is no dependency between the applications then this will result in two independent options according to type A. Contrary, type C represents a compound option where the investment in the IIS creates the opportunity for the first follow-on investment. This one offers the possibility to implement a second follow-on opportunity. Again it is advisable for the valuation to reduce the n to two stages. Finally, there remains "Type D: Staged/Staged" where a reduction to either type B or type C is recommended. Once more the simplification depends on the emphasis of the investments. To summarise, the different investment types lead to the following option types:

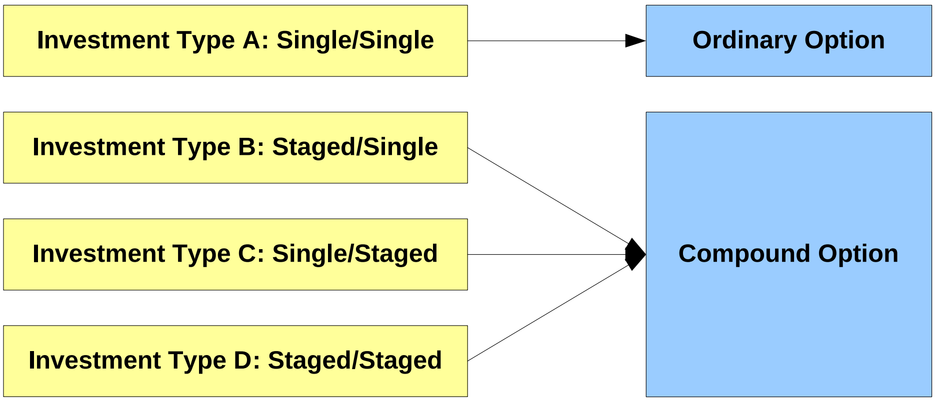


Figure 35: Relationship between investment type and option type
Source: Own representation

According to section 3.2 the basic valuation model for an ordinary option is the Black-Scholes model (for details see 3.2.5) whereas Geske's model is the fundamental one for compound options (see 3.2.9).

4.2.2.3.2 Step 5: Estimation of the Volatility

The estimation of the volatility is a very crucial part as it has a strong impact on the option value. Moreover, it is a compulsory part to estimate the volatility of the underlying asset because every valuation model needs this input parameter. In case of a real option on a follow-on investment the option's underlying equals the present value its benefits. If the valuation accounts for uncertain implementation costs then the costs' volatility will be determined too. For both values a Monte-Carlo simulation according to Godinho [GoPe06] provides the leading estimation. Copeland and Antikarov [CoAn02, CoAn05], as well as Herath and Park [HePa02], also propose a Monte-Carlo simulation which is easier but more imprecise than Godinho's. Miller and Park [MiPa04] suggest a more straightforward approach which does not use a Monte-Carlo simulation, but the so-called modified scenario approach. Section 3.3.2.3 describes all these methods in sufficient detail. The input data for the estimation is available from the NPV scenarios of step three.

4.2.2.3.3 Step 6: Estimation of the remaining Input-Parameters

The remaining input-parameters (see 3.3.3) are obtainable from the cash flow statement which is used for the NPV calculation in step three. The present value ($t = 0$) of the application's benefits equals the value of the underlying asset. Additionally, the computation uses the costs of the implementation as the option's exercise price. For that it is necessary to discount the costs to the option's maturity date, i.e. the date when the implementation starts. Moreover, the interest rate of an Austrian government bond serves as the risk-free rate of interest. Finally, the estimation of the correlation between benefits and costs remains if the implementation costs are uncertain. As the existing literature provides no mathematical methods it is the common way to ask the management for an approximation of the value.

4.2.2.3.4 Step 7: Calculation of the Option Value

After the execution of the steps five and six it is possible to calculate the option's value. The option type of step four and the extent of complexity determine the valuation model in the following way:

Option Type	Complexity	Closed Form Model	Binomial Tree Model
Ordinary Option	Fixed costs, no convenience yields	Black-Scholes [BlSc73] see Section 3.2.5	Cox, Ross, Rubinstein [CoRR79] see Section 3.2.11.1
Ordinary Option	Uncertain costs, no convenience yields	Margrabe [MaWi78] see Section 3.2.8	Rubinstein [RuMa91] see Section 3.2.11.3
Ordinary Option	Fixed costs, convenience yields	Merton [MeRo73] see Section 3.2.6	Cox, Ross, Rubinstein [CoRR79] see Section 3.2.11.2
Ordinary Option	Uncertain costs, convenience yields	McDonald, Siegel [McSi85] see Section 3.3.2.1	Rubinstein [RuMa91] see Section 3.2.11.3
Compound Option	Fixed costs, no convenience yields	Geske [GeRo79a] see Section 3.2.9	Adaptation of Cox, Ross, Rubinstein see Section 3.3.2.2
Compound Option	Uncertain costs, no convenience yields	Carr [CaPe88] see Section 3.3.2.1	Adaptation of Rubinstein see Section 3.3.2.2
Compound Option	Fixed costs, convenience yields	Adaptation of Geske see Section 3.2.9	Adaptation of Cox, Ross, Rubinstein see Section 3.3.2.2
Compound Option	Uncertain costs, convenience yields	Carr [CaPe88] see Section 3.3.2.1	Adaptation of Rubinstein see Section 3.3.2.2

Table 12: Overview about the applied valuation models
Source: Own representation

The table presents every method and the conditions of its application. It lists each model of the previous sections except of Fischer's formula. Instead of this formula the study applies Margrabe's because here it is easier to estimate the necessary input parameters. However, the important point is that there exists a closed form solution as well as a binomial tree approximation for each of the eight cases. This is a precondition for the process model in order to be able to examine assumption three. To summarise, the models from above lead to eight results for the value of one option: 2 * 4 outcomes because of the four different degrees of complexity and the two different models, namely closed form solution and binomial tree approximation.

4.2.2.3.5 Step 8: American Option Valuation

After the calculation of the eight possible European option values the next step is to determine the American option value. In this context it is important to keep in mind that an American value exceeds the European value only in the existence of dividends or convenience yields respectively. The binomial model yields the American option value as described in chapter 3.2.11.2 and allows for investigating assumption nine.

4.2.2.3.6 Step 9: Sensitivity Analysis

Step nine uses a sensitivity analysis in order to study the influence of the input parameters volatility, correlation coefficient, PV benefits, and PV costs. During the analysis several option values are calculated by modifying each of these variables separately while all others stay constant. Afterwards it is possible to compare the different results and thus to measure the influence of the change in the option value.

4.2.2.4 Phase 4: Investment Decision

Though the final phase consists of only one step it is the most import one. The goal of step nine is to reach a decision concerning the investment in the IIS with the following decision rule:

IF Strategic NPV > 0 THEN invest in the IIS, ELSE do not invest ***Strategic NPV = NPV of the IIS + ∑value of option i***

Consequently this step uses the results of step two and those of step seven in order to decide about the investment in the IIS. As step seven yields more than one option value the question arises which of these the relevant one is. This refers to assumption six which analyses the different values and their impact on the investment decision. If there is no clear decision, i.e. the strategic NPV is near zero then the results of the sensitivity analysis provide useful assistance. They help to interpret the situation by investigating whether or not an input parameter's adjustment leads to another investment decision.

4.3 Case Study Selection

4.3.1 Strategy

In the beginning of this research the goal was defined to perform a multiple case research by investigating four cases. The main obstacle for that was the company's compliance to cooperate and its permission to use the collected data in an anonymous form. Beside these issues the most important criteria is the existence of the research objective, i.e. an investment in an EAI-tool, ERP-tool, or a data warehouse. First, the effort was to find case studies where the investment decision in the IIS was not made in the past but will be a task to be performed in the future. Unfortunately, no case existed where it was possible to receive the company's authorization to gather all relevant data. Hence, this research investigates cases where the company made the investment decision in the past and re-calculates this investment using the real options approach. In such an ex-ante examination the comparison between the calculated result and the actual value is a potential area of investigation. A precondition for the determination of the actual value is that the useful life of the systems under consideration is over. Otherwise, i.e. if the system is still in use it is impossible to take all generated cash flows into consideration. Moreover, an IIS represents an investment in IT-infrastructure und thus has quite a long useful life. As the investment decisions of the cases are quite current this precondition is not fulfilled by any one of them. On the other hand this represents an advantage because if the IIS is not in use any more then it will be very difficult to collect the relevant data to a sufficient extent.

However, the intention of the case selection is to find similar cases where each of them allows for the application of the process model for the valuation. Benbasat et al. [BeGM87] call this "literal replication" compared to theoretical replication where contradictory outcomes are expected. Hence, the following four case studies will be presented in Part 5:

- Case Study 1: "EAI and Position Keeping"
  Implementation of an EAI-tool in a large-sized Austrian bank
- Case Study 2: "Data Warehouse and Bank Controlling"
  Implementation of a data warehouse in a medium-sized German bank
- Case Study 3: "EAI and Process Automation"
  Implementation of an EAI-tool in a large-sized Austrian bank
- Case Study 4: "EAI and Order Management"
  Implementation of an EAI-tool in a subsidiary of a large-sized Austrian bank

The presentation of each case follows a structure which contains three main issues. First, a description of the initial situation is offered including a depiction of the project and of the company's investment decision. The second part addresses the valuation according to the process model. Finally, the third part interprets the results of the second part and investigates the assumptions.

4.3.2 Data and Data Collection

This research follows typical case study researches because it uses interviews and documentary materials to collect the relevant data [BeGM87, MyMi97, and YiRo03]. As a special form of interviews the study uses unstructured interviews, i.e. the set of questions is not fixed in advance. Instead the interviewer prepares only a guideline and is thus able to adjust its questions to the progress of the interview. For the purpose of ensuring a structured and consistent course of action the data collection follows the subsequent process:

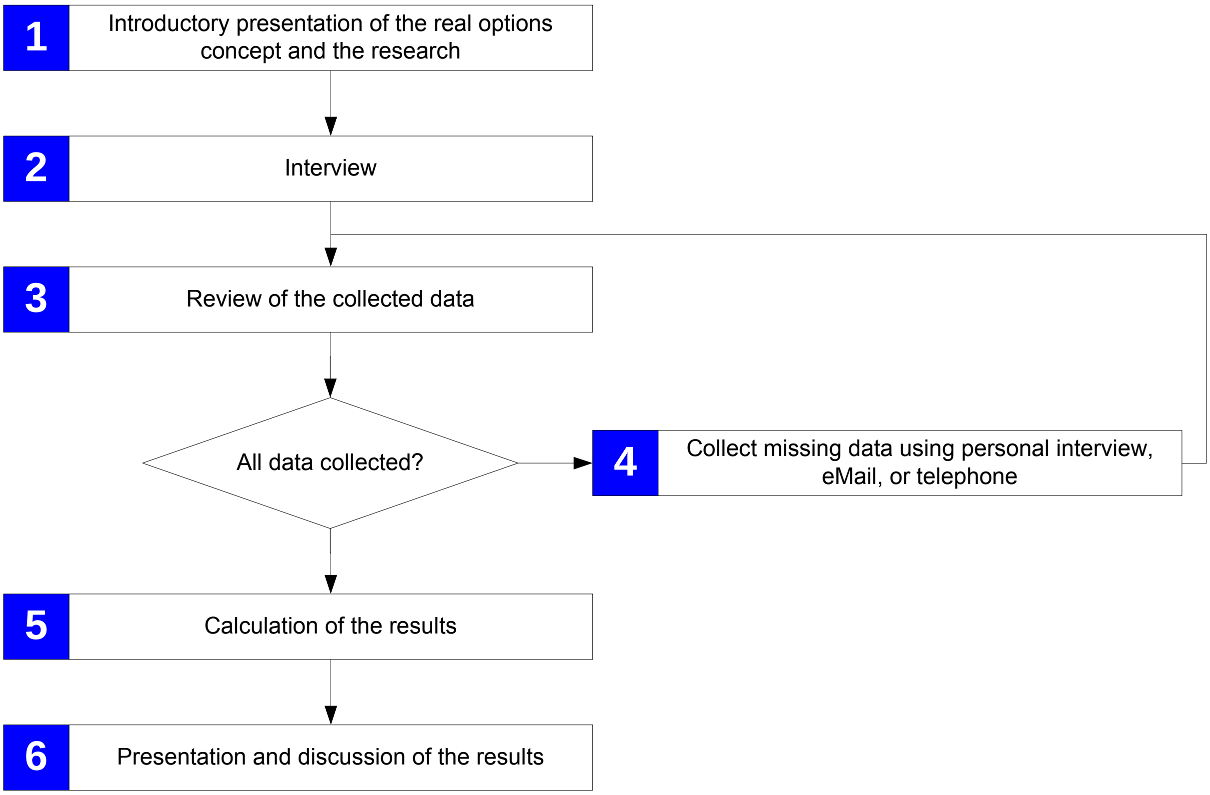


Figure 36: Process of data collection
Source: Own representation

The initial meeting with the contact person of the company consists of two parts. In the beginning a short presentation is held explaining some relevant aspects of the real options concept. Moreover, the presentation also includes an introduction of the research project. Afterwards the interview starts using the guideline for the collection of the desired data. Both documents, the presentation as well as the guideline are available in the appendix. They are in German as all interview partners had German as their native language. After the interview the review of the gathered data starts. As the interview partner will not be able to provide all information in the first meeting additional information exchange via email and telephone is necessary. In cases where it is not possible to solve all open issues over these media additional interviews come up. When the data is complete the next step is the calculation according to the process model from above. Finally, there is another meeting with the decision maker of the company including a presentation and a discussion of the results. Another topic in this appointment is the examination of assumption 10 concerning the applicability of the process model.

With exception of the first case study each of them follows the described process. The data of the first case study origins from a research of Axmann [AxTh06]. He investigates an integration investment with various methods including a real options analysis. The author was involved in this study and offered some advice for Axmann's valuation. However, the present research improves and deepens the original analysis of the case and thus provides new insights. For detailed information about the collected data the interested reader is referred to the appendix where the data of all four case studies is shown in detail.

Part 5: EMPIRICAL ANALYSIS

For the various calculations of the empirical part this thesis applies two tools, namely Microsoft® Excel and Decisioneering's® Crystal Ball. The first one is well known but does not allow for Monte-Carlo simulations which are necessary for some volatility-estimation techniques. Hence, the latter one, which is an Excel-based risk simulation software, is used for them. Furthermore, Crystal Ball offers also great assistance in conducting sensitivity analyses. The regressions for the volatility estimation according to Godinho are again performed with Microsoft® Excel.

5.1 Case Study 1: "EAI and Position Keeping"

5.1.1 Description of the Initial Situation

The first case study takes place in the department for trading and investment banking in a large-sized Austrian bank. As the business activities grew in a remarkable extent during the last years many systems which have been either developed or bought are in action. The system landscape consists of many applications and even more interfaces between them. Moreover, various technologies, environments, and proprietary formats in the heterogeneous interfaces are in use. The result of these issues is a blown up effort for the maintenance of the existing and the implementation of new systems. Additionally, the high level of complexity raises the operating costs even more. In order to improve this situation, the bank investigates the implementation of an EAI-tool. The three most important goals of this project are:

1. Improvement of the process- and workflow-management, regardless of the involved systems and interfaces
2. Decrease in the operative efforts, which are mostly a result of the proprietary connections between the systems
3. Introduction of standards and thus a reduction of the various technologies which are in use

For the purpose of simplifying the process- and workflow-management, a tool which allows for the design, construction, controlling, and documentation of the processes and the workflows, is necessary. This tool lessens the need to interfere manually and the obligation for continuous monitoring. Beside the second point, which aims at eliminating most of the point-to-point connections the last objective deals with benefits derived from synergy effects of standardised components. A more detailed description of the case is available in [AxTh06].

The bank investigates three different products using a NPV-analysis. Product A has a NPV of -60,000 EUR, product B one of -212,000 EUR, and also product C shows a negative NPV of -116,000 EUR. Based on these results the company decides not to undertake the investment.

5.1.2 Calculation

This section shows the application of the process model from chapter 4.2.2 on the case. The subsequent section uses these results for an examination of the assumptions.

5.1.2.1 Phase 1: Option Identification

The EAI-tool enables a follow-on project which has the goal to consolidate the various position-keeping-systems of the banking group [AxTh06]. After the project there should only remain one system in the headquarters, uniting all other systems of the subsidiaries. In order to reach this goal the following fundamental tasks have to be completed:

1. Disassembly of the subsidiaries' systems
2. Migration of the positions of these systems in the central one
3. Replacement of the existing interfaces, especially to data users from controlling, risk management, and local reporting to the national bank

The next figure gives a description of the relationship between the IIS, i.e. the EAI-tool and the option on the follow-on opportunity:

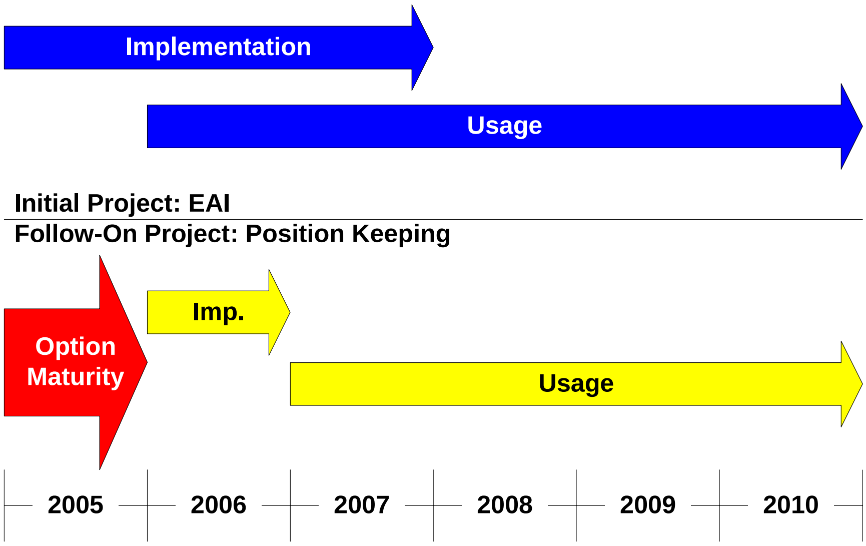


Figure 37: Case Study 1 – Timeline
Source: Own representation, based on [AxTh06]

The figure shows that the implementation of the IIS starts in January 2005 and lasts three years. After one year it will already be possible to use the first parts and the company considers whether or not to undertake the follow-on investment. Thus, the maturity of the option equals one year.

5.1.2.2 Phase 2: NPV Calculation

This study concentrates on Product A as it clearly has the best NPV in comparison to the other products. The subsequent two tables show the NPV calculation for the IIS as well as for the follow-on investment. Whereas the IIS has a negative value of -60,000 EUR the value of the investment in the position-keeping-systems is clearly positive with 373,000 EUR. Hence, if the bank had considered the follow-on opportunity using the NPV method then it would have come to a positive investment decision for the IIS because of an overall NPV of 310,000 EUR. Moreover, the NPV analyses of the best and the worst scenario yield to values of 1,320,263 EUR and -578,180 EUR, respectively. Detailed data concerning the scenarios is available in the appendix.

Input Parameters of the calculation					
Discount rate	7.00%	Project-Start	01.01.2005	Start of use	01.01.2006
		Project-End	31.12.2007	End of use	31.12.2010

Year	**2005**	**2006**	**2007**	**2008**	**2009**	**2010**
Period	**0**	**1**	**2**	**3**	**4**	**5**
Project Expenses						
Hardware	-120,000					
Costs for the server						
Software	-112,000	-168,000	-281,000			
Payment in three instalments						
Internal personnel	-284,783	-330,090	-87,540			
1,070 man-days á 660 EUR in t=0 (increase of 2% per year)						
External personnel	-24,510	-33,600	-25,710			
100 man-days á approx. 840 EUR						
Operating Expenses						
Software license	-101,000	-101,000	-101,000	-101,000	-101,000	-101,000
approx. 18% of the investment costs of 561,000 EUR						
Benefits						
Savings in personnel				415,544	431,395	453,063
1,855 man-days á 687 EUR in t=3 (increase of 2% per year)						
Savings in licence-costs and maintenance			155,000	310,000	310,000	310,000
Cash-flow of period t	-642,293	-632,690	-340,250	624,544	640,395	662,063
Discounted cash-flow (t = 0)	-642,293	-591,299	-297,187	509,814	488,555	472,042
NPV	**-60,369**					

Table 13: Case Study 1 – NPV calculation IIS
Source: Own representation, based on [AxTh06]

Input Parameters of the calculation								
Discount rate	7.00%	Project-Start	01.01.2006			Start of use	01.01.2007	
		Project-End	31.12.2006			End of use	31.12.2010	

Year	**2005**	**2006**	**2007**	**2008**	**2009**	**2010**
Period	**0**	**1**	**2**	**3**	**4**	**5**
Project expenses						
Hardware		-28,000				
8 * 3,500						
Software A		-228,000				
240,000 with 5% discount						
Software B		-67,500				
Personnel		-1,568,987				
2,330 man-days á 673.385						
Operating expenses						
Hardware			-3,360	-3,360	-3,360	-3,360
12% of investment costs						
Software A			-42,750	-42,750	-42,750	-42,750
18.75% of investment consts						
Personnel (increase of 2% per year)			-297,636	-303,589	-309,661	-315,854
2 1/6 FTE (433 man-days á 673.385)						
Benefits						
Reduced effort for coordination (increase of 2% per year)			240,398	245,206	250,111	255,113
1.75 FTE						
Reduced effort for reconciliation (increase of 2% per year)			34,343	35,029	35,730	36,445
0.25 FTE						
Reduced effort for maintenance of interfaces (increase of 2% per year)			497,968	507,928	518,086	528,448
3 5/8 FTE						
Savings in licenses			234,000	234,000	234,000	234,000
Cash-flow of period t	0	-1,892,487	662,963	672,465	682,156	692,041
Discounted cash-flow (t = 0)	0	-1,768,679	579,058	548,931	520,414	493,416
NPV	**373,139**					

Table 14: Case Study 1 – NPV calculation follow-on investment
Source: Own representation, based on [AxTh06]

5.1.2.3 Phase 3: Option Valuation

Phase three starts with the identification of the option type according to section 4.2.2.3. As there is only one investment in the IIS and also only one follow-on investment, an ordinary option is the subject of the valuation. The next step is to estimate the volatility of the underlying asset and of the exercise price using the different approaches. The underlying asset consists of all cash flows of the categories "benefits" and "operating expenses" of the years two until five. The sum of all cash flows of the category "project expenses" yields the exercise price. Given this definition and using the data of the NPV calculation of the three scenarios the input parameters for the estimation according to the modified scenario approach are as follows:

Asset					
Scenario	**PV_0**	**$PV_1 + C_1$**	**Probability**	**z-Score**	**r**
Best		3,037,282	90.00%	1.28	
Normal	2,141,819	2,291,746	50.00%	0.00	6.77%
Worst		1,546,210	10.00%	-1.28	

Exercise Price					
Scenario	**PV_0**	**$PV_1 + C_1$**	**Probability**	**z-Score**	**r**
Best		1,624,601	10.00%	-1.28	
Normal	1,768,679	1,892,487	50.00%	0.00	6.77%
Worst		2,164,863	90.00%	1.28	

Table 15: Case Study 1 – Input parameters modified scenario approach
Source: Own representation

Moreover, the NPV scenarios also provide the parameters for the Monte-Carlo simulation. There it is necessary to define for every uncertain cash flow the type of its distribution, the cash flow's mean, and its standard deviation. It is assumed that the variables are normally distributed with the subsequent characteristics:

Asset						
Parameter	**Normal**	**Best**	**Worst**	**Mean**	**Stddev**	**Stddev %**
Software A	-228,000	-216,000	-240,000	-228,000	9,364	-4.11%
Personnel (operating)	-2 1/6	-2	-2 1/3	-2 1/6	0.1301	-6.00%
Effort coordination	1 3/4	2 1/2	1	1 3/4	0.5852	33.44%
Effort reconciliation	1/4	1/2	0	1/4	0.1951	78.03%
Effort maintenance	3 5/8	4	3 1/4	3 5/8	0.2926	8.07%

Exercise Price						
Parameter	**Normal**	**Best**	**Worst**	**Mean**	**Stddev**	**Stddev %**
Software A	-228,000	-216,000	-240,000	-228000	9,364	-4.11%
Personnel	-1,568,987	-1,313,101	-1,829,363	-1,570,483	199,669	-12.71%

Table 16: Case Study 1 – Input parameters Monte-Carlo simulation
Source: Own representation

The standard deviation is calculated based on the best scenario and not on the worst scenario as the option's value depends mostly on the underlying asset's upside potential:

$$\sigma = \frac{\text{Value Best Scenario} - \text{Mean}}{1.28}$$

However, as the normal scenario is nearly the mean, the differences had been minimal if the worst scenario would have been used. Furthermore, the simulation also accounts for the autocorrelation of each variable between two subsequent years. In order to keep the simulation as simple as possible the autocorrelation coefficient is estimated with 0.9 for all variables. Finally, the various approaches lead to the following volatility estimates:

Approach which is used for the estimation	Volatility asset	Volatility exercise price
Modified Scenario Analysis (Normal : Worst)	30.71%	10.49%
Modified Scenario Analysis (Best : Worst)	26.34%	11.20%
Modified Scenario Analysis (Best : Normal)	21.98%	11.91%
Simulation Herath and Park	19.89%	15.33%
Simulation Copeland and Antikarov	14.05%	10.82%
Simulation/Regression Godinho	13.12%	10.79%

Table 17: Case Study 1 – Volatility estimates
Source: Own representation

Detailed information concerning the regression of Godinho's approach is available in the appendix. All simulations were run with 50,000 trials but with exception of the one suggested by Copeland and Antikarov all other methods lead to an upward bias in comparison to Godinho's technique. The estimates of the volatility of the exercise price are quite the same as there are only cash flows of one period under consideration. After identifying the remaining inputs the valuation according to Black-Scholes yields the following result:

Parameter	Value
Underlying asset (S)	2,141,819
Exercise price (K)	1,892,487
Years to maturity (t)	1.00
Risk-free rate (rf)	3.50%
Volatility (σ)	13.12%

Option Value according to Black-Scholes: 328,679 EUR

Table 18: Case Study 1 – Option valuation according to Black-Scholes
Source: Own representation

The risk-free interest rate origins from the Austrian government bond AT0000386198. Surprisingly, the option value of 328,679 EUR is lower than the NPV, which equals 373,139.

This is an effect caused by discounting the costs with different interest rates: in the option valuation the discount rate is 3.5% leading to 1.82m EUR at t=0 whereas in the NPV analysis the discount rate equals 7% which leads to 1.77m EUR. This discrepancy of 58,716 EUR explains the spread of 44,460 EUR in the results. However, the binomial tree approximation yields 327,926 EUR which is very near the closed form solution:

Input Parameters	
Risk-free rate of interest (rf)	3.56%
Discount factor	1.0059
Upward factor (u)	1.0550
Downward factor (d)	0.9478
Risk-neutral probability (p)	54.12%
Risk-neutral probability (1-p)	45.88%

Price movement of the underlying asset

Year	0	2/12	4/12	6/12	8/12	10/12	1
							2,953,877
						2,799,781	
					2,653,725		2,653,725
				2,515,287		2,515,287	
			2,384,071		2,384,071		2,384,071
		2,259,701		2,259,701		2,259,701	
	2,141,819		2,141,819		2,141,819		2,141,819
		2,030,086		2,030,086		2,030,086	
			1,924,182		1,924,182		1,924,182
				1,823,803		1,823,803	
					1,728,660		1,728,660
						1,638,480	
							1,553,005

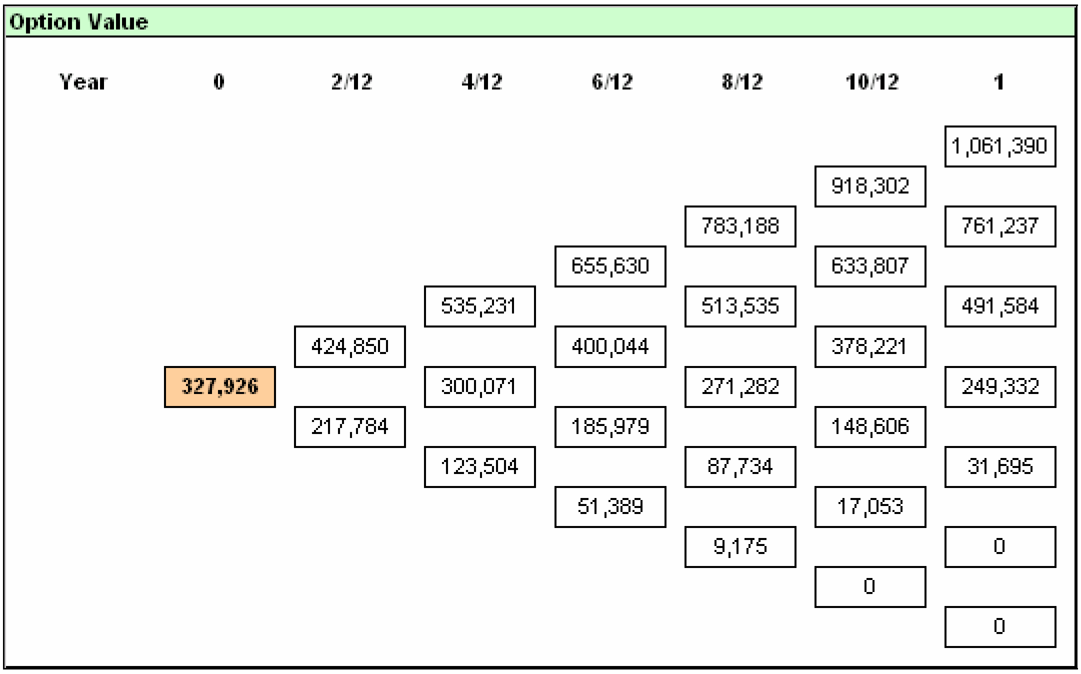


Figure 38: Case Study 1 – Binomial tree approximation Black-Scholes
Source: Own representation

For the binomial tree it is necessary to convert the continuous risk-free rate of interest (3.50%) into a discrete one (3.56%) using the relationship $r_{discrete} = e \wedge r_{continuous} - 1$. Moreover, the discount factor has to take into account the periods per year: discount factor = $(1 + r_f) \wedge (1 /$ number of periods per year). The option maturity of one year consists of only 6 periods, i.e. the steps in the tree show an interval of two months.

So far the exercise price was not uncertain but fix. This changes with the application of Margrabe's formula for the valuation of an exchange option. There the option holder has the opportunity to exchange one uncertain asset (the costs) against another one (the benefits):

Parameter	Value
Asset 1: Costs (C)	1,768,679
Asset 2: Benefits (B)	2,141,819
Years to maturity (t)	1.00
Volatility Costs (σ_C)	10.79%
Volatility Benefits (σ_B)	13.12%
Correlation (ρ_{BC})	0.50

Option Value according to Margrabe: 378,899 EUR

Table 19: Case Study 1 – Option valuation according to Margrabe
Source: Own representation

The result of 378,999 EUR shows that the option value increases about 15% due to discounting the costs with 7% instead of 3.5%. Again, the binomial tree technique leads to a very good approximation of 377,547. The tree of the ratio asset 2 / asset 1, as well as the one for the option value, is available in the appendix.

The next step in the option valuation is to account for convenience yields. Similar to a dividend of a stock, the convenience yield of an asset is not available for the option holder. For the calculation, the following modifications are necessary. First, the value of the underlying asset must include the years zero and one, i.e. it is assumed that the follow-on project is already over and generates its cash flows. As these cash flows are available for the option holder only after the option's maturity they represent the convenience yield. In other words they refer to the loss which occurs due to a start of the project in period one. A consideration of the additional theoretical cash flows of the years zero and one increases the underlying asset's value by 1,255,401 EUR from 2,141,819 EUR to 3,397,220 EUR. That amount, i.e. -36.95% represents the desired convenience yield. A conversion into a continuous rate leads to -46.13% ($\ln(r_{discrete}+1) = r_{continuous}$). The subsequent figure provides more details of this calculation.

Year	2005	2006	2007	2008	2009	2010
Period	**0**	**1**	**2**	**3**	**4**	**5**
Operating expenses						
Hardware	-3,360	-3,360	-3,360	-3,360	-3,360	-3,360
12% of investment costs						
Software A	-42,750	-42,750	-42,750	-42,750	-42,750	-42,750
18.75% of investment consts						
Personnel (increase of 2% per year)	-286,079	-291,800	-297,636	-303,589	-309,661	-315,854
2 1/6 FTE (433 man-days á 673.385)						
Benefits						
Reduced effort for coordination (increase of 2% per year)	231,063	235,685	240,398	245,206	250,111	255,113
1.75 FTE						
Reduced effort for reconciliation (increase of 2% per year)	33,009	33,669	34,343	35,029	35,730	36,445
0.25 FTE						
Reduced effort for maintenance of interfaces (increase of 2% per year)	478,631	488,204	497,968	507,928	518,086	528,448
3 5/8 FTE						
Savings in licenses	234,000	234,000	234,000	234,000	234,000	234,000
Cash-flow of period t	644,515	653,648	662,963	672,465	682,156	692,041
Cash-flow of period t discounted at t = 0	644,515	610,886	579,058	548,931	520,414	493,416
Present value of the underlying asset at t = 0	**3,397,220**					

Sum of cash-flows which are not available for the option holder (t=0)	-1,255,401
Discrete convencience yield (CFs in % of PV)	**-36.95%**
Continuous convencience yield (CFs in % of PV)	**-46.13%**

Table 20: Case Study 1 – Calculation convenience yield
Source: Own representation

After determining the convenience yield the option value is calculated using Merton's formula:

Parameter	Value
Underlying asset (S)	3,397,220
Exercise price (K)	1,892,487
Years to maturity (t)	1.00
Risk-free rate (rf)	3.50%
Volatility (σ)	13.12%
Convenience yield (q)	46.13%

Option Value according to Merton: 328,679 EUR

Table 21: Case Study 1 – Option valuation according to Merton
Source: Own representation

The result is the same compared to that of the valuation with the Black-Scholes formula. The binomial tree approximation produces 327,926 EUR, i.e. also the same outcome as before:

Input Parameters	
Risk-free rate of interest (rf)	3.56%
Discount factor	1.0059
Upward factor (u)	1.0550
Downward factor (d)	0.9478
Risk-neutral probability (p)	54.12%
Risk-neutral probability (1-p)	45.88%
Convenience yield per year (q)	36.95%
Convenience yield per period	7.40%

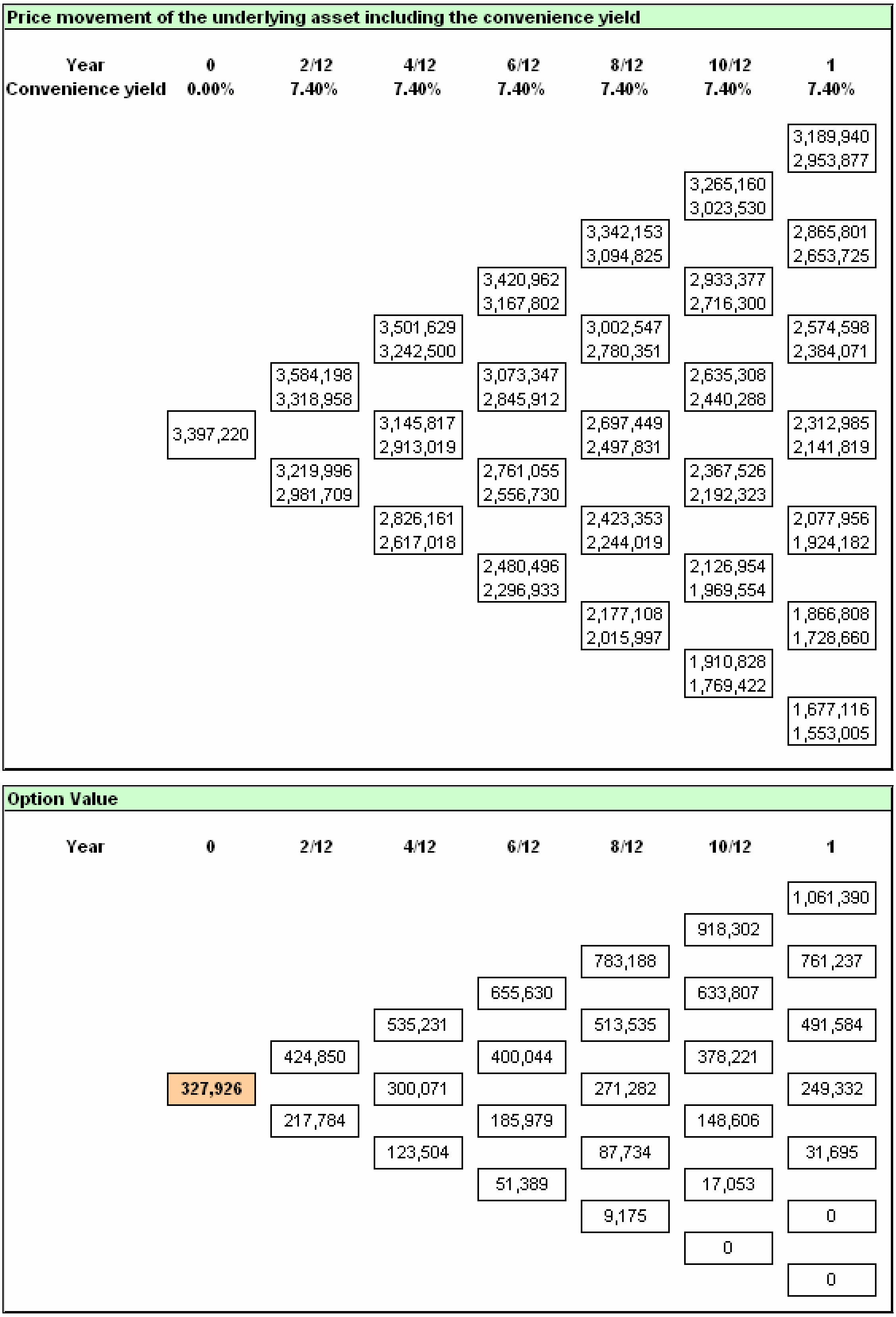


Figure 39: Case Study 1 – Binomial tree approximation Merton
Source: Own representation

According to the process model it remains that valuation accounting for a convenience yield and uncertain costs. Using the formula of McDonald and Siegel leads to a result of 378,899 EUR which is the same as before gained by Margrabe's formula:

Parameter	Value
Asset 1: Costs (C)	1,768,679
Asset 2: Benefits (B)	3,397,220
Years to maturity (t)	1.00
Volatility Costs (σ_C)	10.79%
Volatility Benefits (σ_B)	13.12%
Correlation (ρ_{BC})	0.50
Convenience yield Asset 1 (δ)	46.13%
Convenience yield Asset 2 (η)	0%

Option Value according to McDonald and Siegel: 378,899 EUR

Table 22: Case Study 1 – Option valuation according to McDonald and Siegel
Source: Own representation

The binomial tree with six periods shows a poor approximation of 444,884 EUR, i.e. a deviation of +17.41%. If the number of periods is enlarged to 16 or even 72 then the results of 401,009 EUR (+5.84%) and 381,882 EUR (+0.79%) are more accurate. The appendix provides more details about the binomial tree valuation.

Step eight of the process model deals with the valuation of the investment as an American option. The goal is to determine whether or not it makes sense to start the follow-on project not in January 2006 but later. The useful life of the IIS ends in December 2010 and the follow-on system needs one year for its implementation. Thus, the option's maturity date is 01/01/2009 in order to ensure at least one year of use. The time needed for implementing the system eliminates an exercise of the option before 01/01/2006. With the purpose of preventing such an early exercise of the American option the convenience yield is set to 0% during the first year. With this information it is possible to set up the cash flow model and to calculate the convenience yield of every year:

Year	2005	2006	2007	2008	2009	2010
Period	**0**	**1**	**2**	**3**	**4**	**5**
Operating expenses						
Hardware			-3,360	-3,360	-3,360	-3,360
12% of investment costs						
Software A			-42,750	-42,750	-42,750	-42,750
18.75% of investment consts						
Personnel (increase of 2% per year)			-297,636	-303,589	-309,661	-315,854
2 1/6 FTE (433 man-days á 673.385)						
Benefits						
Reduced effort for coordination (increase of 2% per year)			240,398	245,206	250,111	255,113
1.75 FTE						
Reduced effort for reconciliation (increase of 2% per year)			34,343	35,029	35,730	36,445
0.25 FTE						
Reduced effort for maintenance of interfaces (increase of 2% per year)			497,968	507,928	518,086	528,448
3 5/8 FTE						
Savings in licenses			234,000	234,000	234,000	234,000
Cash-flow of period t	0	0	662,963	672,465	682,156	692,041
Cash-flow of period t discounted at t = 0	0	0	579,058	548,931	520,414	493,416
Present value of the underlying asset at t = 0	**2,141,819**					

Start of project at		t=1	t=2	t=3	t=4	
Lost cash-flows		periods 0,1	periods 0-2	periods 0-3	periods 0-4	
Present value of the underlying asset before deduction of the lost cash-flows		2,141,819	2,141,819	1,562,761	1,013,830	
Sum of cash-flows which are lost, i.e. not available for the option holder (t=0)		0	579,058	548,931	520,414	
Discrete convencience yield (CFs in % of PV)		**0.00%**	**27.04%**	**35.13%**	**51.33%**	

Table 23: Case Study 1 – Calculation convenience yield (American option)
Source: Own representation

The binomial tree approximation for the European option without dividends and maturity of one year serves as a starting point. It has produced an outcome of 327,926 EUR by using six periods for the approximation. The American valuation enlarges the tree up to 24 periods, i.e. 18 additional periods for the years two, three, and four. Moreover, the exercise price has to be adjusted using the risk-free rate of interest. At t = 0 it equals 1,892,487 EUR but at t = 4 2,102,005 EUR. The option holder has to make the following decisions:

- At the end of the tree: “Exercise or no exercise?”
  i.e. max (Value of the underlying – Convenience yield – Exercise price; 0)
- In the middle of the tree: “Early exercise or continue?”
  i.e. max (Value of the underlying – Convenience yield – Exercise price;
  $p * \text{Option value}_{UP} + (1 - p) * \text{Option Value}_{DOWN}$)

The resulting American option value of 327,926 EUR equals exactly the European one. Details of the calculation are available in the appendix. The important point is that it generates no benefit when there exists the possibility to postpone the implementation. The next figure reflects this fact and shows the option’s exercise strategy. If there is no early exercise at t = 1 then for such cases there will not be any exercise later on. Hence, the tree could be cut after t = 1 without any change in the outcome and the American option value equals the European one.

Exercise Strategy

Year	0	2/12	4/12	6/12	8/12	10/12	1	1 2/12	1 4/12	1 6/12	1 8/12	1 10/12	2
	No	No	No	No	No	No	Early	Early	Early	Early	Early	Early	Early
		No	No	No	No	No	Early	Early	Early	Early	Early	Early	Early
			No	No	No	No	Early	Early	Early	Early	Early	Early	Early
				No	No	No	Early	Early	Early	Early	Early	Early	Early
					No	No	Early	Early	Early	Early	No	No	No
						No	No	No	No	No	No	No	No
							No	No	No	No	No	No	No
								No	No	No	No	No	No
									No	No	No	No	No
										No	No	No	No
											No	No	No
												No	No
													No

2 2/12	2 4/12	2 6/12	2 8/12	2 10/12	3	3 2/12	3 4/12	3 6/12	3 8/12	3 10/12	4
Early	Early	Early	Early	Early	Early	Early	Early	Early	No	No	No
Early	Early	Early	Early	Early	Early	Early	Early	No	No	No	No
Early	Early	Early	Early	Early	Early	No	No	No	No	No	No
Early	Early	Early	No	No	No	No	No	No	No	No	No
No	No	No	No	No	No	No	No	No	No	No	No
No	No	No	No	No	No	No	No	No	No	No	No
No	No	No	No	No	No	No	No	No	No	No	No
No	No	No	No	No	No	No	No	No	No	No	No
No	No	No	No	No	No	No	No	No	No	No	No
No	No	No	No	No	No	No	No	No	No	No	No
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No	No	No	No	No	No	No	No	No	No	No	No
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		No	No	No	No	No	No	No	No	No	No
			No	No	No	No	No	No	No	No	No
				No	No	No	No	No	No	No	No
					No	No	No	No	No	No	No
						No	No	No	No	No	No
							No	No	No	No	No
								No	No	No	No
									No	No	No
										No	No
											No

Figure 40: Case Study 1 – Exercise strategy American option

Source: Own representation

When regarding the exercise price as uncertain there is nearly no change in the outcome. The binomial tree with six periods per year shows an option value of 381,454 EUR, i.e. +1.03% compared to the original approximation of 377,547 EUR. Using 16 periods per year leads to almost the same result (380,025 EUR; +0.66%) as the holder of the option exercises very soon after t=1. Thus, there exist cases where an exercise later than in t = 1 is desirable but the benefits of them are very small.

The next section provides a summary and interpretation of all results of the valuation but before, there remains step ten of the process model leading to a positive investment decision:

	Black-Scholes (Fixed Costs)	**Margrabe (Uncertain Costs)**
NPV of the IIS	-60,369	-60,369
Option Value	328,679	378,899
Strategic NPV	*268,311*	*318,531*
Investment decision	☑	☑

Table 24: Case Study 1 – Investment decision
Source: Own representation

5.1.3 Interpretation of the Results

As expected in assumption one, the NPV of the IIS without consideration of the follow-on opportunity is negative. For an examination of the other assumptions the next table shows all option values of the previous chapter:

<table>
<tr><th rowspan="2">Type</th><th rowspan="2">Result Closed form</th><th colspan="3">Binomial Tree Approximation</th><th colspan="2">American Option Valuation</th></tr>
<tr><th>Periods Per year</th><th>Result</th><th>Difference</th><th>Result</th><th>Difference</th></tr>
<tr><td>Fixed Costs,
No Convenience Yield</td><td>328,679</td><td rowspan="3">6</td><td>327,926</td><td>-0.23%</td><td rowspan="2">327,926</td><td rowspan="2">-0.23%</td></tr>
<tr><td>Fixed Costs,
Convenience Yield</td><td>328,679</td><td>327,926</td><td>-0.23%</td></tr>
<tr><td>Uncertain Costs,
No Convenience Yield</td><td>378,899</td><td>377,547</td><td>-0.36%</td><td rowspan="2">381,454</td><td rowspan="2">0.67%</td></tr>
<tr><td rowspan="6">Uncertain Costs,
Convenience Yield</td><td rowspan="6">378,899</td><td>6</td><td>444,884</td><td>17.41%</td></tr>
<tr><td>12</td><td>410,033</td><td>8.22%</td><td>-</td><td>-</td></tr>
<tr><td>16</td><td>401,009</td><td>5.84%</td><td>380,025</td><td>0.30%</td></tr>
<tr><td>24</td><td>391,854</td><td>3.42%</td><td>-</td><td>-</td></tr>
<tr><td>48</td><td>383,884</td><td>1.32%</td><td>-</td><td>-</td></tr>
<tr><td>60</td><td>381,882</td><td>0.79%</td><td>-</td><td>-</td></tr>
</table>

Table 25: Case Study 1 – Results option valuation
Source: Own representation

Quite surprisingly the NPV of the follow-on investment (373,139 EUR) is higher than the option value with fixed implementation costs. This is due to the different discount rates and has already been explained in the previous chapter. Contrary, the comparison between the NPV and the option value with uncertain costs shows a slightly higher option value (+1,5%). The binomial tree valuations with only six periods provide very good approximations. This picture changes when a convenience yield comes into consideration. In this case an enlargement to 16 periods again yields a good estimation. However, for European options it is not necessary to use a complex valuation technique which accounts for convenience yields. With an appropriate input-model this technique shows the same outcome as that one which does not account for convenience yields. The difference between regarding the implementation costs as uncertain (378,899 EUR) or fixed (328,679 EUR) is significant with 13%. Otherwise, the difference is not high enough that the investment decision would be another one. Moreover, the NPV of the follow-on investment would also lead to a clearly positive investment decision.

In order to investigate assumption eight a sensitivity analysis explores the influence of the volatility of the benefits on the option value. While all other input parameters stay constant the changing volatility results in the following outcomes:

Volatility (%)	A) Black Scholes	Deviation	B) Margrabe	Deviation	BinTree BS	Difference to A)	BinTree Margrabe	Difference to B)
5	314,443	95.67%	374,499	98.84%	314,423	-0.01%	374,282	-0.06%
10	319,152	97.10%	375,767	99.17%	319,779	0.20%	375,075	-0.18%
11	321,664	97.87%	376,507	99.37%	322,206	0.17%	375,431	-0.29%
12	324,700	98.79%	377,483	99.63%	324,816	0.04%	375,838	-0.44%
13	328,214	99.86%	378,726	99.95%	327,575	-0.19%	377,255	-0.39%
13.12	**328,679**	**100.00%**	**378,899**	**100.00%**	**327,926**	**-0.23%**	**377,547**	**-0.36%**
14	332,156	101.06%	380,260	100.36%	330,457	-0.51%	379,690	-0.15%
15	336,477	102.37%	382,103	100.85%	333,441	-0.90%	382,282	0.05%
16	341,132	103.79%	384,266	101.42%	339,104	-0.59%	385,009	0.19%
17	346,081	105.29%	386,749	102.07%	345,398	-0.20%	387,850	0.28%
18	351,290	106.88%	389,548	102.81%	351,822	0.15%	390,788	0.32%
19	356,727	108.53%	392,655	103.63%	358,353	0.46%	393,809	0.29%
20	362,365	110.25%	396,055	104.53%	364,977	0.72%	396,901	0.21%
22	374,155	113.84%	403,676	106.54%	378,449	1.15%	403,260	-0.10%
25	392,856	119.53%	416,902	110.03%	399,079	1.58%	413,128	-0.91%
30	425,905	129.58%	442,664	116.83%	434,178	1.94%	440,765	-0.43%
35	460,446	140.09%	471,689	124.49%	469,775	2.03%	475,568	0.82%
40	495,883	150.87%	502,864	132.72%	505,606	1.96%	510,757	1.57%

Table 26: Case Study 1 – Sensitivity analysis volatility
Source: Own representation

First, it is again observable that the binomial tree approximates the results of the closed form solutions very well. Second, the difference between a valuation with fixed and uncertain costs becomes smaller the higher the asset's volatility. This is explained by the fact that the costs' volatility stays constant at a low level and thus has only a minor impact. In general, a change in the volatility has a higher impact when costs are fixed. Moreover, a change in the volatility from 13.12% to 14% increases the option value only by 1%. Thus, the estimation of Copeland and Antikarov would provide nearly the same result as Godinho's. A calculation according to Herath and Park leads to a significant rise of about +10% in the option value according to Black and Scholes. The increase in Margrabe's value is only 5%. This value will rise if also the change in the costs' volatility is considered. The estimates of the modified scenario approach were 22%, 26%, and 31%, i.e. they also produce an upward bias in the option value. However, the bias resulting of a volatility of 22% is +14% / +7% which is quite a good approximation for the fact that no Monte-Carlo analysis is carried out. Another interesting sensitivity analysis considers the correlation coefficient:

Correlation (%)	Margrabe	Deviation	Binomial Tree	Difference to Margrabe
0	394,603	104.14%	395,604	0.25%
5	392,900	103.70%	394,039	0.29%
10	391,212	103.25%	392,432	0.31%
15	389,542	102.81%	390,781	0.32%
20	387,895	102.37%	389,082	0.31%
25	386,277	101.95%	387,329	0.27%
30	384,694	101.53%	385,518	0.21%
35	383,154	101.12%	383,643	0.13%
40	381,667	100.73%	381,695	0.01%
45	380,245	100.36%	379,667	-0.15%
50	**378,899**	**100.00%**	**377,547**	**-0.36%**
55	377,647	99.67%	375,901	-0.46%
60	376,508	99.37%	375,431	-0.29%
65	375,502	99.10%	374,933	-0.15%
70	374,654	98.88%	374,399	-0.07%
75	373,988	98.70%	373,822	-0.04%
80	373,521	98.58%	373,187	-0.09%
85	373,254	98.51%	373,139	-0.03%
90	373,154	98.48%	373,139	0.00%
95	373,139	98.48%	373,139	0.00%
100	373,139	98.48%	373,139	0.00%

Table 27: Case Study 1 – Sensitivity analysis correlation
Source: Own representation

The table shows that the correlation coefficient has only a minor influence on the option value. As the estimation of this parameter is difficult, this is a very useful insight. Furthermore, the binomial tree approximation yields results which are again very close to those of the closed form approach.

The outcome of the American option valuation is nearly the same as the European one, regardless of fixed or uncertain implementation costs. Thus, the earliest start of implementation at t = 1 is the optimal strategy in almost all cases. The benefit of early exercise stays quite small even in case of a higher volatility:

	Fixed Costs			Uncertain Costs				
Volatility (%)	European Value	American Value	Difference	European Value	American 6 periods	Difference	American 24 periods	Difference
5	314,423	314,423	0.00%	374,282	374,274	0.00%	374,757	0.13%
10	319,779	319,779	0.00%	375,075	376,082	0.27%	375,971	0.24%
11	322,206	322,206	0.00%	375,431	378,042	0.70%	376,881	0.39%
12	324,816	324,816	0.00%	375,838	379,652	1.01%	378,301	0.66%
13	327,575	327,575	0.00%	377,255	381,250	1.06%	379,830	0.68%
13.12	**327,926**	**327,926**	**0.00%**	**377,547**	**381,454**	**1.03%**	**380,025**	**0.66%**
14	330,457	330,457	0.00%	379,690	382,954	0.86%	381,592	0.50%
15	333,441	333,786	0.10%	382,282	384,773	0.65%	383,491	0.32%
16	339,104	339,890	0.23%	385,009	386,685	0.44%	385,468	0.12%
17	345,398	346,413	0.29%	387,850	388,672	0.21%	389,039	0.31%
18	351,822	353,063	0.35%	390,788	393,376	0.66%	392,716	0.49%
19	358,353	359,820	0.41%	393,809	398,834	1.28%	396,411	0.66%
20	364,977	366,669	0.46%	396,901	404,294	1.86%	400,463	0.90%
25	399,079	402,035	0.74%	413,128	432,252	4.63%	425,065	2.89%
30	434,178	438,950	1.10%	440,765	462,725	4.98%	457,082	3.70%
35	469,775	476,751	1.49%	475,568	499,488	5.03%	491,926	3.44%
40	505,606	520,700	2.99%	510,757	545,611	6.82%	532,750	4.31%

Table 28: Case Study 1 – Sensitivity analysis volatility American option
Source: Own representation

Moreover, also when changing the correlation coefficient the American option value is not significantly different from the European one:

Correlation (%)	Binomial Tree European	American 6 periods	Difference	American 24 periods	Difference
0	395,604	402,019	1.62%	398,698	0.78%
5	394,039	399,244	1.32%	396,689	0.67%
10	392,432	396,363	1.00%	394,738	0.59%
15	390,781	393,364	0.66%	392,708	0.49%
20	389,082	390,241	0.30%	390,591	0.39%
25	387,329	388,308	0.25%	388,379	0.27%
30	385,518	387,041	0.40%	386,061	0.14%
35	383,643	385,727	0.54%	384,480	0.22%
40	381,695	384,361	0.70%	383,063	0.36%
45	379,667	382,938	0.86%	381,575	0.50%
50	**377,547**	**381,454**	**1.03%**	**380,025**	**0.66%**
55	375,901	379,883	1.06%	378,519	0.70%
60	375,431	378,043	0.70%	376,881	0.39%
65	374,933	374,963	0.01%	375,754	0.22%
70	374,399	374,414	0.00%	374,932	0.14%

Correlation (%)	Binomial Tree European	American 6 periods	Difference	American 24 periods	Difference
75	373,822	383,410	2.57%	374,068	0.07%
80	373,187	376,080	0.78%	373,553	0.10%

Table 29: Case Study 1 – Sensitivity analysis correlation American option
Source: Own representation

The table shows no correlation coefficients above 80% which is due to the fact that the number of periods (six per year or 24 per year) is not sufficient for the other cases, i.e. these results are not usable. Another sensitivity analysis concerns the value of the underlying asset as well as the exercise price. Previously, it was mentioned that the carelessness of using the simple valuation model produces a deviation of 13% in the option value. The subsequent table shows that a lack of care for the estimation of the PV results in a larger bias:

	PV Benefits (t=0)	PV Costs (t=0)	Option Value Black Scholes	Option Value Margrabe	Differ-ence	Deviation Black Scholes	Deviation Margrabe
Original Values	**2,141,819**	**-1,768,679**	**328,679**	**378,899**	**-13.25%**	**100.00%**	**100.00%**
Benefits +5%	2,248,910	-1,768,679	427,962	482,387	-11.28%	130.21%	127.31%
Benefits -5%	2,034,728	-1,768,679	236,714	280,156	-15.51%	72.02%	73.94%
Costs -5%	2,141,819	-1,857,113	252,910	298,929	-15.39%	76.95%	78.89%
Costs +5%	2,141,819	-1,680,246	411,657	463,515	-11.19%	125.25%	122.33%

Table 30: Case Study 1 – Sensitivity analysis present values
Source: Own representation

A change of +/-5% of the benefits causes nearly a modification of +/-30% in the option value whereas an adjustment of +/-5% of the costs leads to a deviation of +/-25%. Consequently, it is very important to spend enough time on the estimation of the cash flows and the determination of the discount rate. This insight holds valid also for the valuation of the American option:

	PV Benefits (t=0)	PV Costs (t=0)	American Option Value			
			Fixed Costs	Deviation	Uncertain Costs	Deviation
Original Values	**2,141,819**	**-1,768,679**	**327,926**	**100.00%**	**380,025**	**100.00%**
Benefits +5%	2,248,910	-1,768,679	428,807	130.76%	482,799	127.04%
Benefits -5%	2,034,728	-1,768,679	239,173	72.94%	282,342	74.30%
Costs -5%	2,141,819	-1,857,113	255,271	77.84%	301,112	79.23%
Costs +5%	2,141,819	-1,680,246	412,410	125.76%	463,904	122.07%

Table 31: Case Study 1 – Sensitivity analysis present values American option
Source: Own representation

Finally, a summary of all outcomes leads to the following judgements concerning the assumptions:

#	Assumptions	Support (++, +, ~, –, – –)
1	Negative NPV of IIS	++
2	NPV smaller than option value	––
3	Binomial tree approximation sufficient	++
4	Model complexity – no convenience yields	++
5	Model complexity – uncertain implementation costs	++
6	Equal investment decision – Simple model vs. complex model	++
7	Different investment decision – NPV vs. option value	––
8	Volatility estimation according to Godinho	–
9	Rough estimation of the correlation coefficient	++
10	Minor benefit of a postponed implementation of the follow-on opportunity	++
11	Small estimation error of the PV has less influence than the model complexity	––

Table 32: Case Study 1 – Overview about the examination of the assumptions
Source: Own representation

5.2 Case Study 2: “Data Warehouse and Bank Controlling”

5.2.1 Description of the Initial Situation

The second case study investigates a data warehouse project in a medium-sized German bank. This bank introduces a new core-banking system which has the effect that the existing data warehouse cannot be used any more. Moreover, it is also necessary to replace the existing software for the reporting to the German national bank with a new one. This is a result of the change in the core-banking system but also of the new regulatory requirements according to Basel II. The bank has made the decision that an external partner will provide the new reporting software. All in all, the data warehouse project has to complete the following major tasks:

1. To ensure the reporting according to Basel II from 01/01/2007 onwards, i.e. to make the necessary data available
2. To guarantee the data supply to the new software for the reporting to the national bank
3. To store all relevant data of the new core-banking system as well as other systems
4. To provide all relevant data for the existing controlling applications and also for some new controlling applications

In the past, a consultant company assisted the bank to implement the old data warehouse and the existing controlling applications. For the new project this company is again the bank’s contractual partner. It is possible to divide the data in the data warehouse into two categories. On the one hand, data which is needed daily, i.e. middle amount of space but high degree of availability and on the other hand, monthly data, i.e. high amount of space but low degree of availability. This fact leads to a split of the data warehouse to two different servers described by the following figure:

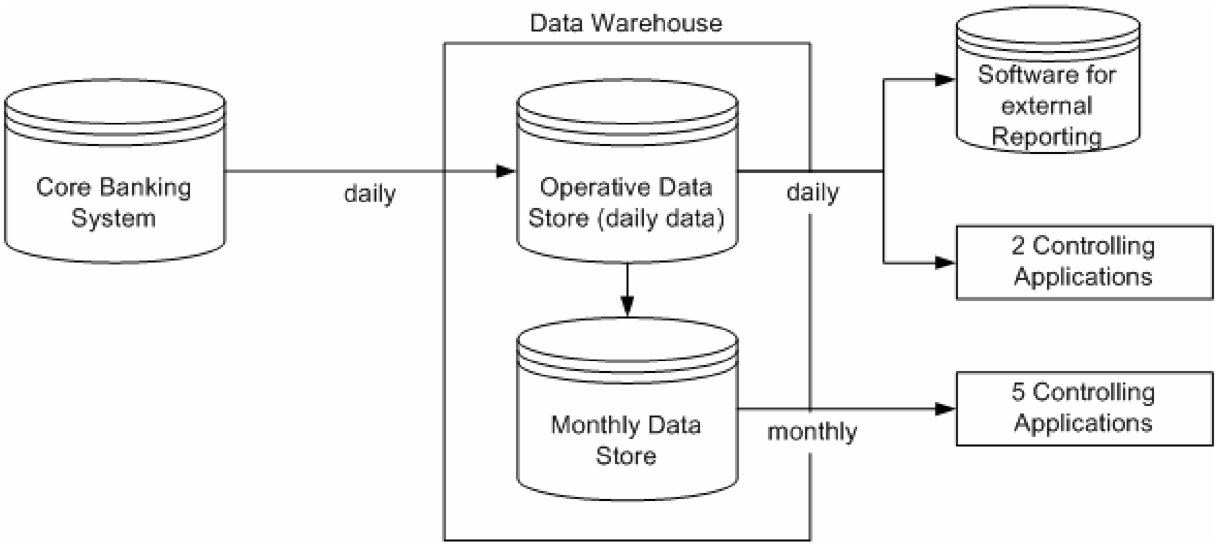

Figure 41: Case Study 2 – System landscape
Source: Own representation

Due to the requirements of Basel II and the new core-banking the project was compulsory system, the bank has made no detailed investment analysis before starting the project.

5.2.2 Calculation

5.2.2.1 Phase 1: Option Identification

The data warehouse generates the possibility to improve the controlling of the bank in the areas of liquidity risk, risk strategy, global limit system, and improvement of the risk processes. Moreover, there is the opportunity to change the approach for the calculation of the regulatory capital requirement from the standardised approach to the internal ratings based approach (IRBA). The benefit of the IRBA is that the bank has to fulfil a lower capital requirement or, in other words, that the bank has more free capital to invest. Both areas, controlling improvements and IRBA introduction, use the data warehouse as data source. The consultant company which assists the bank in the implementation of these projects is the same as for the IIS project. They offered their services for both issues in one contract with a fixed price, i.e. both parts are analysed as only one investment. The project "data warehouse" starts in January 2005 and ends at 30/06/2006, whereas the follow-on project "bank controlling and IRBA" lasts from 01/07/2006 until 31/12/2008. The bank plans a very long useful life of the infrastructure investment, namely until 31/12/2026:

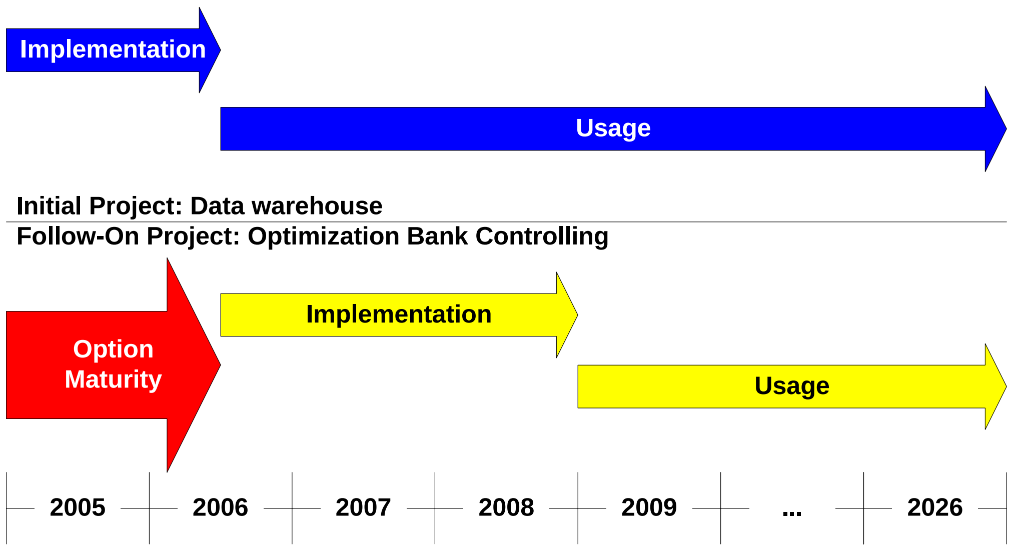

Figure 42: Case Study 2 – Timeline
Source: Own representation

5.2.2.2 Phase 2: NPV Calculation

The following tables show the NPV calculation of the data warehouse as well as the follow-on investment. The investment in the IIS causes high capital outlays of > 5,000,000 EUR in the beginning. As only the replacement of the existing systems was determined as quantitative benefit, they are quite small. The project's main goal is the fulfilment of the reporting requirements and creates no positive cash flows. Even with the long useful life of 20.5 years it is not possible to compensate the high project expenses. This results in a clearly negative NPV of -8,727,618 EUR. Contrary, the NPV of the follow-on investment is obviously positive with 25,503,587 EUR which stems from the savings in the regulatory capital requirements. These savings are so high that not only the best scenario but even the worst scenario has a NPV considerably above zero (32,920,210 EUR and 14,378,652 EUR).[1] Hence, the conclusion is that the follow-on investment justifies the IIS-investment even in the worst case. The subsequent two tables show the NPV analyses of the IIS and the follow-on opportunity in the normal scenario. The other two scenarios are available in the appendix.

[1] The very long useful life has a positive influence on the NPV but even a reduction from 18 years (2009 until 2026) to two years (2009 until 2010) would lead to a positive NPV in the worst case scenario.

Input Parameters of the calculation						
Discount rate	6.00%	Project-Start	01.01.2005		Start of use	01.07.2006
		Project-End	30.06.2006		End of use	31.12.2026

Year	**2005**	**2006**	**2007**	**2008**	**2009**	**...**	**2026**
Period	**0**	**1**	**2**	**3**	**4**		**21**
Project Expenses							
Hardware	-2,500,000	0					
Costs for the server, Workstations, ...							
Software	-44,500	-44,500					
License controlling applications							
Internal personnel	-350,000	-175,000					
5 FTE per year á 70,000 EUR							
External personnel	-1,920,000	-529,505					
1,769 man-days							
Operating Expenses							
External costs for maintenance of controlling applications		-8,010	-16,020	-16,020	-16,020		-16,020
External costs for maintenance of data warehouse		-91,980	-80,980	-69,980	-58,980		-46,980
Internal personnel for maintenance (1 FTE)		-35,000	-70,000	-70,000	-70,000		-70,000
Hardware		-250,000	-500,000	-500,000	-500,000		-500,000
Benefits							
Savings due to the replacement		135,000	270,000	270,000	270,000		270,000
of the existing reporting software							
Savings due to the replacement		45,000	90,000	90,000	90,000		90,000
of the existing data warehouse							
Cash-flow of period t	-4,814,500	-953,995	-307,000	-296,000	-285,000		-273,000
Discounted cash-flow (t = 0)	-4,814,500	-899,996	-273,229	-248,527	-225,747		-80,304
NPV	**-8,727,618**						

Table 33: Case Study 2 – NPV calculation IIS
Source: Own representation

Input Parameters of the calculation					
Discount rate	6.00%	Project-Start	01.07.2006	Start of use	01.01.2009
		Project-End	31.12.2008	End of use	31.12.2026

Year	2005	2006	2007	2008	2009	...	2026
Period	**0**	**1**	**2**	**3**	**4**		**21**
Project expenses							
External personnel		-200.100	-555.278	-266.800			
Internal personnel (8.4 FTE)		-588.000	-588.000	-588.000			
Operating expenses							
Internal personnel (0.6 FTE)					-42.000		-42.000
Benefits							
Savings in the regulatory capital requirements of 52m EUR (return of 6%)					3.120.000		3.120.000
Cash-flow of period t	0	-788.100	-1.143.278	-854.800	3.078.000		3.078.000
Discounted cash-flow (t = 0)	0	-743.491	-1.017.513	-717.707	2.438.064		905.410
NPV	**25.503.587**						

Table 34: Case Study 2 – NPV calculation follow-on investment
Source: Own representation

5.2.2.3 Phase 3: Option Valuation

Analogously to the first case study the subject of valuation is an ordinary option, i.e. the investment in the data warehouse generates the possibility to invest in one follow-on opportunity. Preparing the calculation of the volatilities according to the modified scenario approach, the subsequent input parameters are observable:

Asset					
Scenario	**PV_0**	**$PV_1 + C_1$**	**Probability**	**z-Score**	**r**
Best		37,245,183	90.00%	1.28	
Normal	27,982,297	29,661,235	50.00%	0.00	5.83%
Worst		18,285,313	10.00%	-1.28	

Exercise Price					
Scenario	**PV_0**	**$PV_1 + C_1$**	**Probability**	**z-Score**	**r**
Best		2,349,760	10.00%	-1.28	
Normal	2,478,710	2,627,433	50.00%	0.00	5.83%
Worst		3,043,941	90.00%	1.28	

Table 35: Case Study 2 – Input parameters modified scenario approach
Source: Own representation

The volatility estimates, using a Monte-Carlo simulation, consider three uncertain types of cash flows:

Asset						
Parameter	**Normal**	**Best**	**Worst**	**Mean**	**Stddev**	**Stddev %**
Savings capital requirement	52,000,000	65,000,000	32,500,000	49,833,333	11,834,609	23.75%
Internal personnel	-0.60	-0.50	-0.75	-0.62	0.09	-14.76%

Exercise Price						
Parameter	**Normal**	**Best**	**Worst**	**Mean**	**Stddev**	**Stddev %**
Internal personnel	-8.40	-7.00	-10.50	-8.63	1.27	-14.76%

Table 36: Case Study 2 – Input parameters Monte-Carlo simulation
Source: Own representation

In conformity with the first case study the calculation of the standard deviation uses the mean and the best scenario in order to account for the upside potential of the underlying asset. Assuming an autocorrelation coefficient of 0.9 for all variables the simulation and regression according to Godinho results in an estimation of 14% / 8.64% for the volatility of the asset / exercise price. The only technique which provides similar results compared to the 14% is the modified scenario approach “best vs. normal”:

Approach which is used for the estimation	Volatility asset	Volatility exercise price
Modified Scenario Analysis (Normal : Worst)	37.75%	11.48%
Modified Scenario Analysis (Best : Worst)	27.76%	10.10%
Modified Scenario Analysis (Best : Normal)	17.77%	8.72%
Simulation Herath and Park	28.18%	12.99%
Simulation Copeland and Antikarov	19.93%	9.19%
Simulation/Regression Godinho	14.00%	8.64%

Table 37: Case Study 2 – Volatility estimates
Source: Own representation

All simulations were run with 50,000 trials but even that of Copeland and Antikarov produces an upward bias of nearly 6%. For the volatility estimation of the exercise price it is necessary to consider only one uncertain cash flow and just three periods. This smaller uncertainty leads to quite the same results for all approaches with exception to Herath's and Park's technique. However, using Godinho's estimation leads to an option value of 25,415,544 EUR according to Black and Scholes.

Parameter	Value
Underlying asset (S)	27,982,297
Exercise price (K)	2,705,107
Years to maturity (t)	1.50
Risk-free rate (rf)	3.50%
Volatility (σ)	14.00%

Option Value according to Black-Scholes: 25,415,544 EUR

Table 38: Case Study 2 – Option valuation according to Black-Scholes
Source: Own representation

Thus, the deviation of the NPV to the option value is less than 1% and the option produces no benefit. The reason for that lies in the NPV of the worst case scenario which is still positive. Hence, the option will be exercised in every case at its maturity as shown by the following binomial tree:

Input Parameters	
Risk-free rate of interest (rf)	3.56%
Discount factor	1.0088
Upward factor (u)	1.0725
Downward factor (d)	0.9324
Risk-neutral probability (p)	54.52%
Risk-neutral probability (1-p)	45.48%

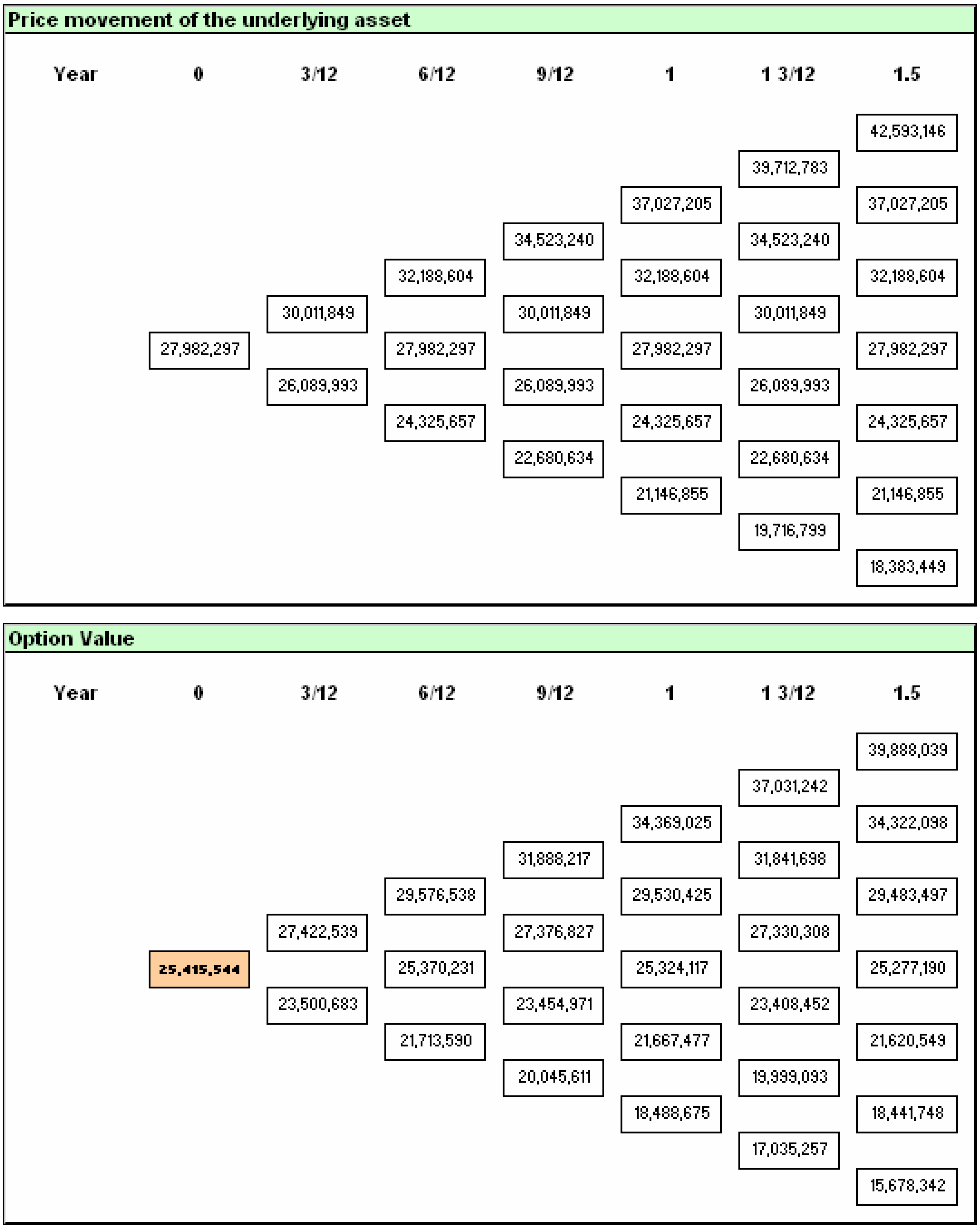

Figure 43: Case Study 2 – Binomial tree approximation Black-Scholes
Source: Own representation

As the tree contains six steps, one singular step represents three months. The approximation provides a very good result, namely exactly the same option value as obtained by the closed form solution.

Applying Margrabe's formula leads to an option valuation with an uncertain exercise price and produces a slightly higher outcome:

Parameter	Value
Asset 1: Costs (C)	2,478,710
Asset 2: Benefits (B)	27,982,297
Years to maturity (t)	1.50
Volatility Costs (σ_C)	8.64%
Volatility Benefits (σ_B)	14.00%
Correlation (ρ_{BC})	0.50

Option Value according to Margrabe: 25,503,587 EUR

Table 39: Case Study 2 – Option valuation according to Margrabe
Source: Own representation

More interesting is the fact that this option value equals exactly the NPV of the normal scenario. The minor difference to the Black/Scholes result stems from the discounting of the exercise price. Whereas the two other methods discount the investment costs with 6% to 2,478,710 EUR, the Black/Scholes formula uses the risk-free interest rate of 3.5%. Hence, the exercise price at t=1.5 of 2,705,107 EUR reduces to only 2,566,753 EUR which explains the difference of -88,043 EUR in the option value. However, the binomial tree technique again leads to an exact approximation which is illustrated in the appendix.

As already depicted in the first case study, the consideration of a convenience yield requires a modification of the PV calculation of the underlying asset. First, it is necessary to include the cash flows which would have been available if the use of the system had already been possible from t=0 onwards leading to a PV of 39,287,828 EUR. Second, the convenience yield of -11,305,531 EUR or -28.78% is calculated using these cash flows. Third, a conversion of this value from one and a half years to one year yields -20.25%:

$$\text{Annualized yield} = (\text{PV after loss} / \text{PV before loss})^{1/1.5} - 1$$

Finally, the calculation of the continuous yield produces a value of -22.62% ($\ln(r_{discrete}+1) = r_{continuous}$). The subsequent figure provides more details of these calculations.

Year	2005	2006	2007	2008	2009	...	2026
Period	0	1	2	3	4		21
Operating expenses							
Internal personnel (0.6 FTE)	-42.000	-42.000	-42.000	-42.000	-42.000		-42.000
Benefits							
Savings in the regulatory capital requirements of 52m EUR (return of 6%)	3.120.000	3.120.000	3.120.000	3.120.000	3.120.000		3.120.000
Cash-flow of period t	3.078.000	3.078.000	3.078.000	3.078.000	3.078.000		3.078.000
Cash-flow of period t discounted at t = 0	3.078.000	2.903.774	2.739.409	2.584.348	2.438.064		905.410
Present value of the underlying asset at t = 0	**39.287.828**						

Sum of cash-flows which are not available for the option holder (t=0)	-11.305.531
CFs in % of PV (discrete)	-28,78%
Annualized discrete convenience yield	**-20,25%**
CFs in % of PV (continuous)	-33,93%
Annualized continuous convenience yield	**-22,62%**

Table 40: Case Study 2 – Calculation convenience yield
Source: Own representation

Using the increased PV and the convenience yield in Merton's formula produces the same option value as obtained by the Black Scholes formula:

Parameter	Value
Underlying asset (S)	39,287,828
Exercise price (K)	2,705,107
Years to maturity (t)	1.50
Risk-free rate (rf)	3.50%
Volatility (σ)	14.00%
Convenience yield (q)	22.62%

Option Value according to Merton: 25,415,544 EUR

Table 41: Case Study 2 – Option valuation according to Merton
Source: Own representation

This value is also the result of the binomial tree approximation with six periods for 1.5 years:

Input Parameters	
Risk-free rate of interest (rf)	3.56%
Discount factor	1.0088
Upward factor (u)	1.0725
Downward factor (d)	0.9324
Risk-neutral probability (p)	54.52%
Risk-neutral probability (1-p)	45.48%
Convenience yield per year (q)	20.25%
Convenience yield per period	5.50%

Price movement of the underlying asset including the convenience yield

Year	0	3/12	6/12	9/12	1	1 3/12	1.5
Convenience yield	0.00%	5.50%	5.50%	5.50%	5.50%	5.50%	5.50%
							45,071,516 42,593,146
						44,468,780 42,023,553	
					43,874,104 41,461,577		39,181,709 37,027,205
				43,287,381 40,907,117		38,657,737 36,532,045	
			42,708,504 40,360,071		38,140,772 36,043,506		34,061,564 32,188,604
		42,137,369 39,820,340		37,630,720 35,561,501		33,606,063 31,758,150	
	39,287,828		37,127,489 35,085,941		33,156,653 31,333,452		29,610,504 27,982,297
		36,630,987 34,616,741		32,713,254 30,914,434		29,214,527 27,608,093	
			32,275,783 30,501,019		28,823,844 27,238,894		25,741,095 24,325,657
				28,438,387 26,874,631		25,396,863 24,000,353	
					25,057,233 23,679,399		22,377,328 21,146,855
						22,078,079 20,864,061	
							19,453,128 18,383,449

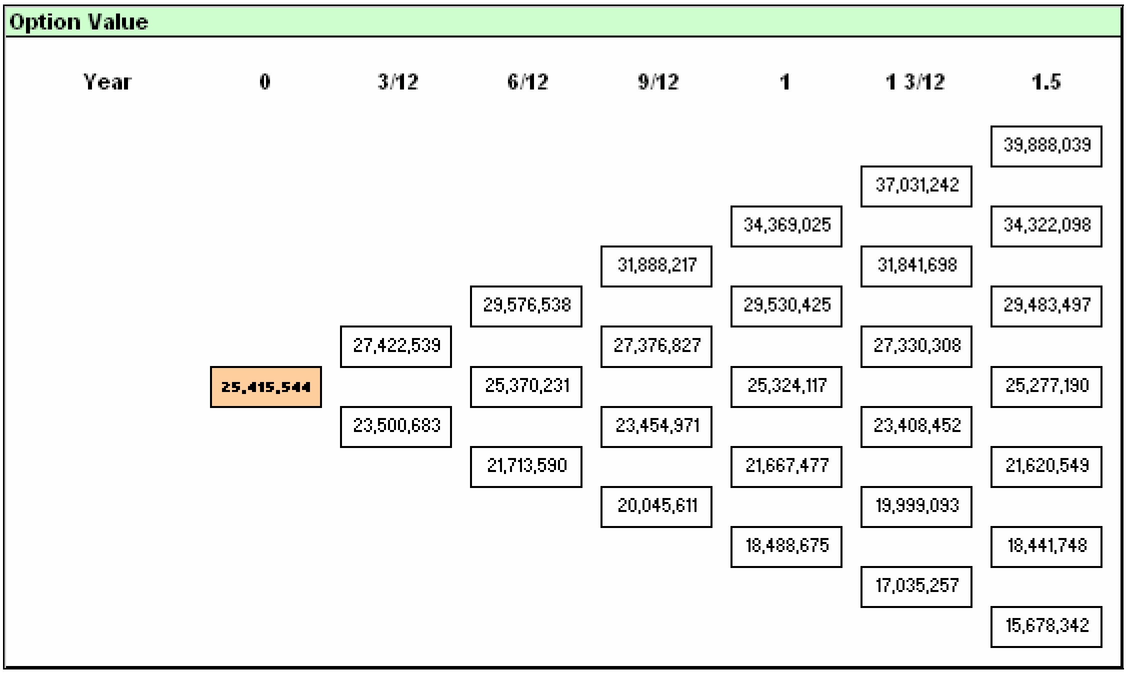

Figure 44: Case Study 2 – Binomial tree approximation Merton
Source: Own representation

The formula of McDonald and Siegel considers not only a convenience yield but also an uncertain exercise price. Like before, the discounting effect increases the option value a little and leads to the same result as Margrabe's formula and the NPV.

Parameter	Value
Asset 1: Costs (C)	2,478,710
Asset 2: Benefits (B)	39,287,828
Years to maturity (t)	1.50
Volatility Costs (σ_C)	8.64%
Volatility Benefits (σ_B)	14.00%
Correlation (ρ_{BC})	0.50
Convenience yield Asset 1 (δ)	22.62%
Convenience yield Asset 2 (η)	0%

Option Value according to McDonald and Siegel: 25,503,587 EUR

Table 42: Case Study 2 – Option valuation according to McDonald and Siegel
Source: Own representation

Contrary to the first case study, the binomial tree approximation generates already with only six periods a very good approximation of 26,016,651 EUR (+2.01%). An enlargement to 12/16 periods improves the result even more with a decreasing deviation of 1.03%/0.78%.

An analysis whether or not it makes sense to postpone the start of the implementation is carried out with the American option valuation. Given the 31/12/2006 as the end of the application's useful life and an implementation time of 2.5 years, the last possible exercise date is 01/07/2023. These 18.5 years time to maturity ensure an useful life of at least one year.

The earliest start of implementation is 01/07/2006, i.e. before this date an early exercise is not possible. The cash flow model representing this information, as well as the calculation of the convenience yield, is available in the following figure.

Year	2005	2006	2007	2008	2009	2010	2011	2012	2013
Period	**0**	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**
Operating expenses									
Internal personnel (0.6 FTE)					-42.000	-42.000	-42.000	-42.000	-42.000
Benefits									
Savings in the regulatory capital requirements of 52m EUR (return of 6%)					3.120.000	3.120.000	3.120.000	3.120.000	3.120.000
Cash-flow of period t	0	0	0	0	3.078.000	3.078.000	3.078.000	3.078.000	3.078.000
Cash-flow of period t discounted at t = 0	0	0	0	0	2.438.064	2.300.061	2.169.869	2.047.046	1.931.175
Present value of the underlying asset at t = 0	**27.982.297**								
Start of project at t=x		1,5	2,5	3,5	4,5	5,5	6,5	7,5	8,5
Lost cash-flows		periods 0-3	periods 0-4	periods 0-5	periods 0-6	periods 0-7	periods 0-8	periods 0-9	periods 0-10
Present value of the underlying asset before deduction of the lost cash-flow		27.982.297	27.982.297	25.544.233	23.244.172	21.074.304	19.027.258	17.096.082	15.274.219
Sum of cash-flows which are lost, i.e. not available for the option holder (t=0)		0	2.438.064	2.300.061	2.169.869	2.047.046	1.931.175	1.821.863	1.718.739
Discrete convencience yield (CFs in % of PV)		**0,00%**	**8,71%**	**9,00%**	**9,34%**	**9,71%**	**10,15%**	**10,66%**	**11,25%**

Year	2014	2015	2016	2017	2018	2019	2020	2021	2022	2023	2024	2025	2026
Period	**9**	**10**	**11**	**12**	**13**	**14**	**15**	**16**	**17**	**18**	**19**	**20**	**21**
Operating expenses													
Internal personnel (0.6 FTE)	-42.000	-42.000	-42.000	-42.000	-42.000	-42.000	-42.000	-42.000	-42.000	-42.000	-42.000	-42.000	-42.000
Benefits													
Savings in the regulatory capital requirements of 52m EUR (return of 6%)	3.120.000	3.120.000	3.120.000	3.120.000	3.120.000	3.120.000	3.120.000	3.120.000	3.120.000	3.120.000	3.120.000	3.120.000	3.120.000
Cash-flow of period t	3.078.000	3.078.000	3.078.000	3.078.000	3.078.000	3.078.000	3.078.000	3.078.000	3.078.000	3.078.000	3.078.000	3.078.000	3.078.000
Cash-flow of period t discounted at t = 0	1.821.863	1.718.739	1.621.452	1.529.672	1.443.087	1.361.402	1.284.342	1.211.643	1.143.060	1.078.358	1.017.319	959.735	905.410
Present value of the underlying asset at t = 0													
Start of project at t=x	9,5	10,5	11,5	12,5	13,5	14,5	15,5	16,5	17,5	18,5			
Lost cash-flows	periods 0-11	periods 0-12	periods 0-13	periods 0-14	periods 0-15	periods 0-16	periods 0-17	periods 0-18	periods 0-19	periods 0-20			
Present value of the underlying asset before deduction of the lost cash-flow	13.555.480	11.934.028	10.404.356	8.961.270	7.599.867	6.315.525	5.103.882	3.960.823	2.882.464	1.865.145			
Sum of cash-flows which are lost, i.e. not available for the option holder (t=0)	1.621.452	1.529.672	1.443.087	1.361.402	1.284.342	1.211.643	1.143.060	1.078.358	1.017.319	959.735			
Discrete convencience yield (CFs in % of PV)	**11,96%**	**12,82%**	**13,87%**	**15,19%**	**16,90%**	**19,19%**	**22,40%**	**27,23%**	**35,29%**	**51,46%**			

Table 43: Case Study 2 – Calculation convenience yield (American option)
Source: Own representation

The binomial tree approximation leads to an American option value of 25,415,544 EUR which is exactly the same as obtained by both the Black Scholes and Merton's formula. Consequently, the possibility of delaying the start of implementation generates no benefits at all. This is reflected by the option's exercise strategy showing an early exercise in every possible case at t = 1.5. The subsequent figure shows this fact whereas the other information of the American option valuation is available in the appendix.

Exercise Strategy

Year	0.0	0.5	1.0	1.5	2.0	2.5	3.0	3.5	4.0	4.5	5.0	5.5	6.0	6.5	7.0	7.5	8.0	8.5
	No	No	No	Early	Early	Early	Early	Early	Early	Early	Early	Early	Early	Early	Early	Early	Early	Early
		No	No	Early	Early	Early	Early	Early	Early	Early	Early	Early	Early	Early	Early	Early	Early	Early
			No	Early	Early	Early	Early	Early	Early	Early	Early	Early	Early	Early	Early	Early	Early	Early
				Early	Early	Early	Early	Early	Early	Early	Early	Early	Early	Early	Early	Early	Early	Early
					Early	Early	Early	Early	Early	Early	Early	Early	Early	Early	Early	Early	Early	Early
						Early	Early	Early	Early	Early	Early	Early	Early	Early	Early	Early	Early	Early
							Early	Early	Early	Early	Early	Early	Early	Early	Early	Early	Early	Early
								Early	Early	Early	Early	Early	Early	Early	Early	Early	Early	Early
									Early	Early	Early	Early	Early	Early	Early	Early	Early	Early
										Early	Early	Early	Early	Early	Early	Early	Early	Early
											Early	Early	Early	Early	Early	Early	Early	Early
												Early	Early	Early	Early	Early	Early	Early
													Early	Early	Early	Early	Early	Early
														Early	Early	Early	Early	Early
															Early	Early	Early	Early
																Early	Early	Early
																	No	No
																		No

9.0	9.5	10.0	10.5	11.0	11.5	12.0	12.5	13.0	13.5	14.0	14.5	15.0	15.5	16.0	16.5	17.0	17.5	18.0	18.5
Early	Early	Early	Early	Early	Early	Early	Early	Early	Early	Early	Early	Early	Early	Early	Early	Early	Early	Early	Yes
Early	Early	Early	Early	Early	Early	Early	Early	Early	Early	Early	Early	Early	Early	Early	Early	Early	Early	Early	Yes
Early	Early	Early	Early	Early	Early	Early	Early	Early	Early	Early	Early	Early	Early	Early	Early	Early	Early	Early	Yes
Early	Early	Early	Early	Early	Early	Early	Early	Early	Early	Early	Early	Early	Early	Early	Early	Early	Early	Early	Yes
Early	Early	Early	Early	Early	Early	Early	Early	Early	Early	Early	Early	Early	Early	Early	Early	Early	Early	Early	Yes
Early	Early	Early	Early	Early	Early	Early	Early	Early	Early	Early	Early	Early	Early	Early	Early	Early	Early	Early	Yes
Early	Early	Early	Early	Early	Early	Early	Early	Early	Early	Early	Early	Early	Early	Early	Early	Early	Early	Early	Yes
Early	Early	Early	Early	Early	Early	Early	Early	Early	Early	Early	Early	Early	Early	Early	Early	Early	Early	Early	Yes
Early	Early	Early	Early	Early	Early	Early	Early	Early	Early	Early	Early	Early	Early	Early	Early	Early	Early	Early	Yes
Early	Early	Early	Early	Early	Early	Early	Early	Early	Early	Early	Early	Early	Early	Early	Early	Early	Early	Early	Yes
Early	Early	Early	Early	Early	Early	Early	Early	Early	Early	Early	Early	Early	Early	Early	Early	Early	Early	Early	No
Early	Early	Early	Early	Early	Early	Early	Early	Early	Early	Early	Early	Early	Early	Early	Early	Early	Early	Early	No
Early	Early	Early	Early	Early	Early	Early	Early	Early	Early	Early	Early	Early	Early	Early	Early	Early	Early	No	No
Early	Early	Early	Early	Early	Early	Early	Early	Early	Early	Early	Early	Early	Early	Early	Early	Early	No	No	No
Early	Early	Early	Early	Early	Early	Early	Early	Early	Early	Early	Early	Early	Early	Early	Early	No	No	No	No
Early	Early	Early	Early	Early	Early	Early	Early	Early	Early	Early	Early	Early	No	No	No	No	No	No	No
No	No	No	No	No	No	No	No	No	No	No	No	No	No	No	No	No	No	No	No
No	No	No	No	No	No	No	No	No	No	No	No	No	No	No	No	No	No	No	No
No	No	No	No	No	No	No	No	No	No	No	No	No	No	No	No	No	No	No	No
	No	No	No	No	No	No	No	No	No	No	No	No	No	No	No	No	No	No	No
		No	No	No	No	No	No	No	No	No	No	No	No	No	No	No	No	No	No
			No	No	No	No	No	No	No	No	No	No	No	No	No	No	No	No	No
				No	No	No	No	No	No	No	No	No	No	No	No	No	No	No	No
					No	No	No	No	No	No	No	No	No	No	No	No	No	No	No
						No	No	No	No	No	No	No	No	No	No	No	No	No	No
							No	No	No	No	No	No	No	No	No	No	No	No	No
								No	No	No	No	No	No	No	No	No	No	No	No
									No	No	No	No	No	No	No	No	No	No	No
										No	No	No	No	No	No	No	No	No	No
											No	No	No	No	No	No	No	No	No
												No	No	No	No	No	No	No	No
													No	No	No	No	No	No	No
														No	No	No	No	No	No
															No	No	No	No	No
																No	No	No	No
																	No	No	No
																		No	No
																			No

Figure 45: Case Study 2 – Exercise strategy American option
Source: Own representation

The valuation of the American option with an uncertain exercise price does not lead to any new insights. Again, the option value of 25,503,587 EUR is the same as the European one meaning that there is no profit in postponing the start of implementation. For details of the valuation the interested reader is referred to the appendix. The final step of the process model concerns the investment decision which is clearly positive:

	Black-Scholes (Fixed Costs)	**Margrabe (Uncertain Costs)**
NPV of the IIS	-8,727,618	-8,727,618
Option Value	25,415,544	25,503,587
Strategic NPV	16,687,926	16,775,969
Investment decision	☑	☑

Table 44: Case Study 2 – Investment decision
Source: Own representation

5.2.3 Interpretation of the Results

The examination of the first assumption leads to a confirmation as the NPV of the IIS without consideration of the follow-on investment is negative. The following table contains all option values of the previous chapter and helps to investigate the other assumptions:

<table>
<tr><th rowspan="2">Type</th><th rowspan="2">Result Closed form</th><th colspan="3">Binomial Tree Approximation</th><th colspan="2">American Option Valuation</th></tr>
<tr><th>Periods per 1.5 years</th><th>Result</th><th>Differ-ence</th><th>Result</th><th>Differ-ence</th></tr>
<tr><td>Fixed Costs, No Convenience Yield</td><td>25,415,544</td><td rowspan="3">6</td><td>25,415,544</td><td>0.00%</td><td rowspan="2">25,415,544</td><td rowspan="2">0.00%</td></tr>
<tr><td>Fixed Costs, Convenience Yield</td><td>25,415,544</td><td>25,415,544</td><td>0.00%</td></tr>
<tr><td>Uncertain Costs, No Convenience Yield</td><td>25,503,587</td><td>25,503,587</td><td>0.00%</td><td rowspan="4">25,503,587</td><td rowspan="4">0.00%</td></tr>
<tr><td rowspan="3">Uncertain Costs, Convenience Yield</td><td rowspan="3">25,503,587</td><td>6</td><td>26,016,651</td><td>2.01%</td></tr>
<tr><td>12</td><td>25,765,963</td><td>1.03%</td></tr>
<tr><td>16</td><td>25,701,499</td><td>0.78%</td></tr>
</table>

Table 45: Case Study 2 – Results option valuation
Source: Own representation

The difference of 0.35% between the NPV of the follow-on opportunity (25,503,587 EUR) and the option value with fixed costs is very small and stems from the already mentioned different discount rates for the costs. However, in comparison to the option value with fixed costs there is no difference at all. Thus, the option to invest generates no profit which is explained by the NPV of the worst scenario. It is still positive, i.e. an exercise of the option is suitable even in the worst case. The binomial tree with only six periods approximates the results of the closed form solutions with a discrepancy of less than 5%. An increase in the model complexity by considering convenience yields leads to no new insights as these valuations produce the same outcomes as before. Similar, the introduction of uncertain costs generates nearly the same results with a variation of only 0.35%. As there are no disparities in the results between the NPV, the simple option model, and the complex option model, the investment decision is the same for all of these methods.

The sensitivity analysis of the volatility of the benefits shows an interesting result. There is no difference in the option value between a volatility of 5% or 40%. Thus, even with a very high volatility the worst outcome of the underlying at the maturity date still leads to an exercise of the option. As the up and down movements per period are symmetric they compensate each other resulting in an unchanged option value.

Volatility (%)	A) Black Scholes	Deviation	B) Margrabe	Deviation	BinTree BS	Difference to A)	BinTree Margrabe	Difference to B)
5	25,415,544	100.00%	25,503,587	100.00%	25,415,544	0.00%	25,503,587	0.00%
10	25,415,544	100.00%	25,503,587	100.00%	25,415,544	0.00%	25,503,587	0.00%
14	**25,415,544**	**100.00%**	**25,503,587**	**100.00%**	**25,415,544**	**0.00%**	**25,503,587**	**0.00%**
20	25,415,544	100.00%	25,503,587	100.00%	25,415,544	0.00%	25,503,587	0.00%
30	25,415,544	100.00%	25,503,587	100.00%	25,415,544	0.00%	25,503,587	0.00%
40	25,415,545	100.00%	25,503,587	100.00%	25,415,544	0.00%	25,503,587	0.00%

Table 46: Case Study 2 – Sensitivity analysis volatility
Source: Own representation

Conducting the sensitivity analysis of the correlation coefficient generates to the same insight: the lower the coefficient, the higher the volatility. However, this increase is not high enough to change the option value:

Correlation (%)	Margrabe	Deviation	Binomial Tree	Difference to Margrabe
0	25,503,587	100.00%	25,503,587	0.00%
20	25,503,587	100.00%	25,503,587	0.00%
50	**25,503,587**	**100.00%**	**25,503,587**	**0.00%**
70	25,503,587	100.00%	25,503,587	0.00%
100	25,503,587	100.00%	25,503,587	0.00%

Table 47: Case Study 2 – Sensitivity analysis correlation
Source: Own representation

As early exercise at t = 1.5 is the optimal strategy there is no increase in the American option value with higher volatility:

	Fixed Costs			Uncertain Costs		
Volatility (%)	European Value	American Value	Difference	European Value	American Value	Difference
5	25,415,544	25,415,544	0.00%	25,503,587	25,503,587	0.00%
10	25,415,544	25,415,544	0.00%	25,503,587	25,503,587	0.00%
14	**25,415,544**	**25,415,544**	**0.00%**	**25,503,587**	**25,503,587**	**0.00%**
20	25,415,544	25,415,544	0.00%	25,503,587	25,503,587	0.00%
30	25,415,544	25,415,544	0.00%	25,503,587	25,503,587	0.00%
40	25,415,544	25,415,544	0.00%	25,503,587	25,503,587	0.00%

Table 48: Case Study 2 – Sensitivity analysis volatility American option
Source: Own representation

Similar to the results from case study one, a lower/higher correlation coefficient has also no positive/negative impact on the American option value:

Correlation (%)	Binomial Tree European	American Value	Difference
0	25,503,587	25,503,587	0.00%
20	25,503,587	25,503,587	0.00%
50	**25,503,587**	**25,503,587**	**0.00%**
70	25,503,587	25,503,587	0.00%
100	25,503,587	25,503,587	0.00%

Table 49: Case Study 2 – Sensitivity analysis correlation American option
Source: Own representation

As the difference between the underlying asset and the strike price is always positive at the maturity date, the option is exercised in all possible cases. Hence, a modification of the PV of the benefits nearly ends in a 1:1 percentage change in the option value:

	PV Benefits (t=0)	PV Costs (t=0)	Option Value Black Scholes	Option Value Margrabe	Differ-ence	Deviation Black Scholes	Deviation Margrabe
Original Values	27,982,297	-2,478,710	25,415,544	25,503,587	-0.35%	100.00%	100.00%
Benefits +5%	29,381,412	-2,478,710	26,814,659	26,902,702	-0.33%	105.50%	105.49%
Benefits -5%	26,583,182	-2,478,710	24,016,429	24,104,472	-0.37%	94.50%	94.51%
Costs -5%	27,982,297	-2,602,646	25,287,207	25,379,652	-0.36%	99.50%	99.51%
Costs +5%	27,982,297	-2,354,775	25,543,882	25,627,523	-0.33%	100.50%	100.49%

Table 50: Case Study 2 – Sensitivity analysis present values
Source: Own representation

In comparison to the first case study, the deviations in the option value are smaller but still higher than the difference between the results of the simple and the complex model. The PV of the costs is minor in relation to the PV of the benefits and thus its change has only a marginal influence on the option value. This holds true also for the American option value:

	PV Benefits (t=0)	PV Costs (t=0)	American Option Value			
			Fixed Costs	Deviation	Uncertain Costs	Deviation
Original Values	27,982,297	-2,478,710	25,415,544	100.00%	25,503,587	100.00%
Benefits +5%	29,381,412	-2,478,710	26,814,659	105.50%	26,902,702	105.49%
Benefits -5%	26,583,182	-2,478,710	24,016,429	94.50%	24,104,472	94.51%
Costs -5%	27,982,297	-2,602,646	25,287,207	99.50%	25,379,652	99.51%
Costs +5%	27,982,297	-2,354,775	25,543,882	100.50%	25,627,523	100.49%

Table 51: Case Study 2 – Sensitivity analysis present values American option
Source: Own representation

To summarise, the subsequent table gives an overview about the examination of all assumptions:

#	Assumptions	Support (++, +, ~, –, – –)
1	Negative NPV of IIS	++
2	NPV smaller than option value	– –
3	Binomial tree approximation sufficient	++
4	Model complexity – no convenience yields	++
5	Model complexity – uncertain implementation costs	– –
6	Equal investment decision – Simple model vs. complex model	++
7	Different investment decision – NPV vs. option value	– –
8	Volatility estimation according to Godinho	– –
9	Rough estimation of the correlation coefficient	++
10	Minor benefit of a postponed implementation of the follow-on opportunity	++
11	Small estimation error of the PV has less influence than the model complexity	– –

Table 52: Case Study 2 – Overview about the examination of the assumptions
Source: Own representation

5.3 Case Study 3: “EAI and Process Automation”

5.3.1 Description of the Initial Situation

A large-sized Austrian bank plans to implement an EAI tool in order to improve the processes for security orders of internal and external managers of investment funds. The main goal is to reach an automatic and standardised process which leads to a reduced processing time per order and a higher quality level. In detail the process of an order placed by an external fund manager looks as follows:

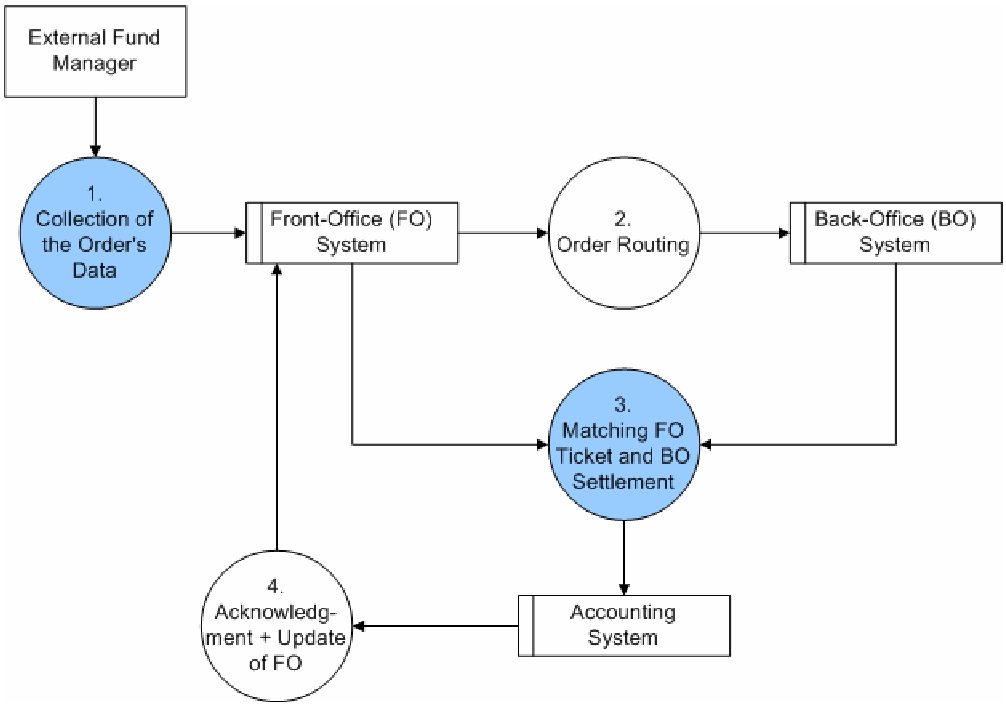

Figure 46: Case Study 3 – Data flow external fund managers
Source: Own representation

First, the external fund manager sends the order’s data via fax, e-mail, or SWIFT to the responsible personnel in the bank who enters the data manually in the front office system. For the order routing from the front office system to the back office system exists an automatic interface which handles the data transfer. The next step concerns the matching between the ticket of the front office system and the settlement of the back office system. This comparison is done by hand using print-outs of the two systems involved. After an update of the accounting system the final process step deals with an update of the front office system with the data of the accounting system. The implementation of the EAI tool allows for two improvements in this process:

1. The orders of fund managers which use SWIFT are transferred automatically to the front office system.
2. The matching between the front office ticket and the back office settlement requires a manual operation only in case of mismatches.

The order process of internal fund managers is quite similar to the one from above:

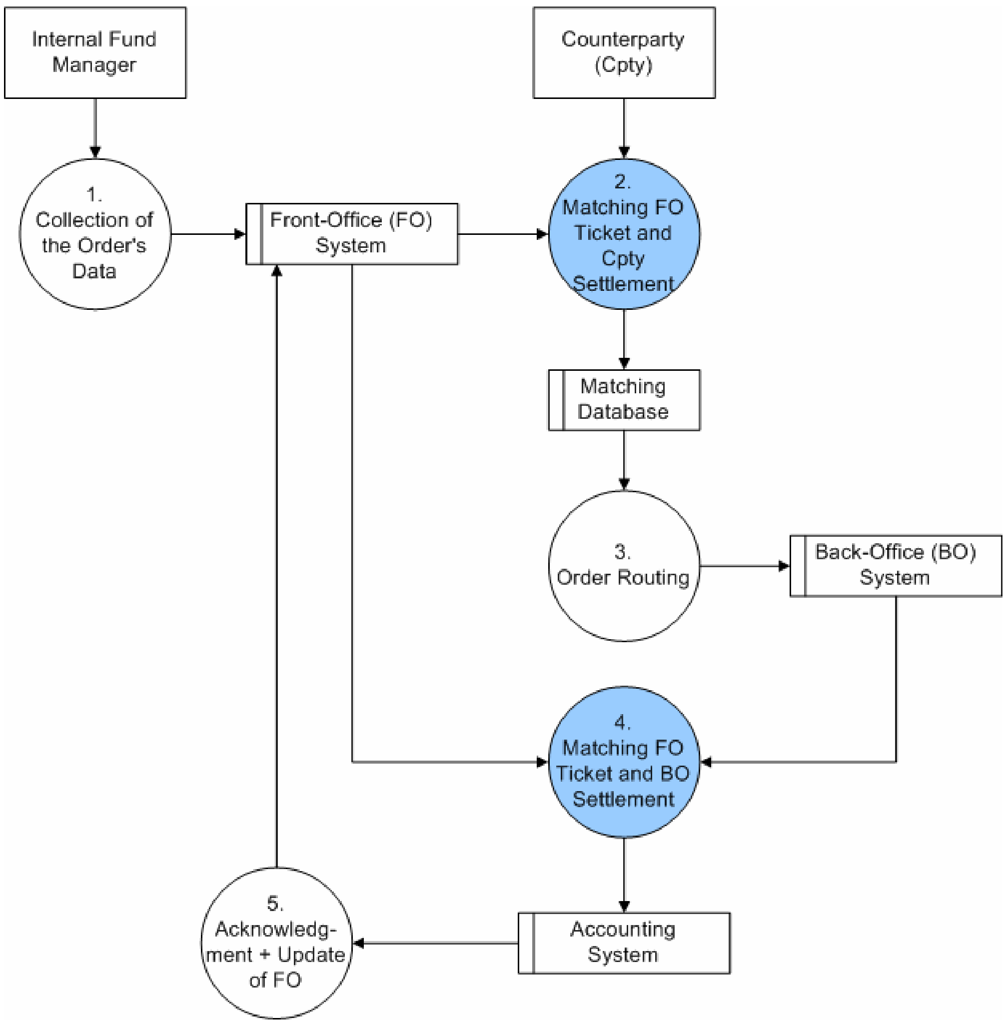

Figure 47: Case Study 3 – Data flow internal fund managers
Source: Own representation

The internal fund manager enters the data of the order directly in the front office system. Then a manual comparison between the front office ticket and the settlement of the counterparty (i.e. the broker) follows. The data transfer between the front office system and the back office system is again automatic. The remaining steps, i.e. the matching of the front office ticket and the back office settlement as well as the update of the front office are the same as above. By implementing the EAI system the following benefits are generated:

1. The matching between the front office ticket and the counterparty settlement requires a manual operation only in case of mismatches.
2. The comparison between the front office ticket and the back office settlement also only causes a manual operation in case of mismatches.

All in all, the described improvements result in a reduction of the processing time per order from t+2 to t+1. The bank made no detailed investment analysis but decided to implement the EAI tool. The benefits of reaching the market standards concerning the processing time, of the compliance with regulations of the bank supervision, and of the automation were regarded as sufficient.

5.3.2 Calculation

5.3.2.1 Phase 1: Option Identification

After implementing the EAI system it is possible to further improve the process for orders of external fund managers as the EAI platform allows for an automatic import in the front office system only when the fund manager uses SWIFT. Contrary, the follow-on project deals with those orders which the bank receives not via SWIFT but by fax or e-mail. The goal is to create and implement a standardized Excel file which enables an automatic processing for the front office system. Hence, this Excel file reduces the manual operations during the order-process and replaces the medias fax and e-mail. Another potential follow-on investment could be the use of the EAI system not only for securities but also for derivatives. However, after an interview with a specialist it was clear that there exists no data at all and that the investment makes no sense due to organizational reasons. Thus, the case study concentrates only on the EAI project and the project “standardised Excel”. The first one starts at 01/11/2004, ends at 31/12/2005 and results in a useful life of the EAI platform of ten years from 01/12/2005 until 01/12/2015. The implementation of the standardized Excel starts at 01/08/2006 and lasts until 31/01/2007 leading to an estimated useful life of eight years and ten months from 01/02/2007 until 01/12/2015. Consequently, the option maturity equals 1.75 years, which is shown in the next figure:

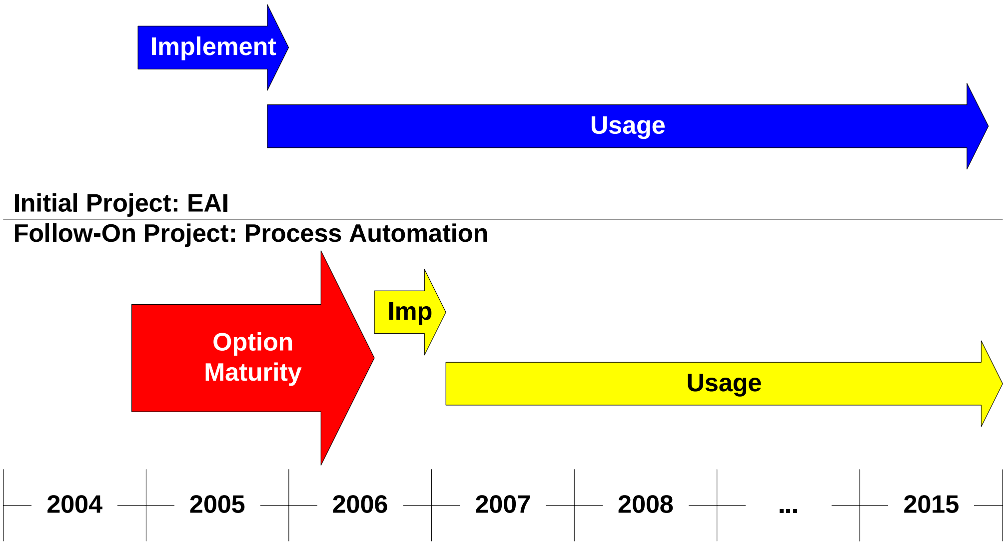

Figure 48: Case Study 3 – Timeline
Source: Own representation

5.3.2.2 Phase 2: NPV Calculation

The NPV calculation of the investment in the EAI application leads to a negative result of -357,922 EUR. Moreover, the NPV of the follow-on investment is also negative with -47,693 EUR. The outcome of the worst scenario is lower (-66,090) and even the best scenario shows only a slightly positive value of 2,824 EUR. Hence, the quite small follow-on investment does not justify the investment in the EAI and the decision relying only on the NPVs would be not to invest. However, for the calculation of the option value this situation is interesting as the normal scenario is negative and even the best scenario has a NPV of nearly zero. This is contrary to the first case study where two out of three scenarios are positive and also to the second one where all 3 scenarios are positive. The subsequent two tables show the NPV analyses of the IIS and the follow-on opportunity in the normal scenario. The other two scenarios are available in the appendix.

Input Parameters of the calculation					
Discount rate	10.19%	Project-Start	01.11.2004	Start of use	01.12.2005
		Project-End	31.12.2005	End of use	01.12.2015

Year	2004	2005	2006	2007	2008	2009	...	2015
Period	0	1	2	3	4	5		11
Project Expenses								
Hardware		-35,000						
Costs for the servers								
Software		-122,000						
Payment in three instalments								
Internal personnel		-124,300						
226 man-days á 550 EUR								
External personnel	-37,584	-392,000						
Operating Expenses								
Software license			-46,800	-46,800	-46,800	-46,800		-46,800
Maintenance Server			-477	-477	-477	-477		-477
Benefits								
Reduced effort for data collection		298	3,915	4,280	4,672	5,091		8,321
Reduced effort for matching of front office system and counterparty settlement		759	9,346	9,885	10,440	11,009		14,678
Reduced effort for matching of front office and back office system		5,364	67,580	70,960	74,507	78,233		104,840
Cash-flow of period t	-37,584	-666,879	33,565	37,848	42,342	47,057		80,561
Discounted cash-flow (t = 0)	-37,584	-605,209	27,644	28,289	28,721	28,967		27,705
NPV	**-357,922**							

Table 53: Case Study 3 – NPV calculation IIS
Source: Own representation

Input Parameters of the calculation					
Discount rate	10.19%	Project-Start	01.08.2006	Start of use	01.02.2007
		Project-End	31.01.2007	End of use	01.12.2015

Year	**2004**	**2005**	**2006**	**2007**	**2008**	**2009**	**...**	**2015**
Period	**0**	**1**	**2**	**3**	**4**	**5**		**11**
Project expenses								
Personnel			-27,500	-13,200				
50 and 24 man-days á 550 EUR								
External personnel			-28,000	-7,000				
Operating expenses								
none (maintenance < 0.5 man-days per year)								
Benefits								
Reduced effort for data collection				2,568	2,920	3,309		6,656
Cash-flow of period t	0	0	-55,500	-17,632	2,920	3,309		6,656
Discounted cash-flow (t = 0)	0	0	-45,710	-13,179	1,980	2,037		2,289
NPV	**-41,693**							

Table 54: Case Study 3 – NPV calculation follow-on investment
Source: Own representation

5.3.2.3 Phase 3: Option Valuation

Again the valuation concerns an ordinary option, i.e. the investment in the EAI system generates the possibility to invest in one follow-on opportunity. In order to calculate the volatility according to the modified scenario approach, the following input parameters are necessary:

Asset					
Scenario	**PV_0**	**$PV_1 + C_1$**	**Probability**	**z-Score**	**r**
Best		50,015	90.00%	1.28	
Normal	19,115	21,063	50.00%	0.00	9.70%
Worst		10,046	10.00%	-1.28	

Exercise Price					
Scenario	**PV_0**	**$PV_1 + C_1$**	**Probability**	**z-Score**	**r**
Best		46,903	10.00%	-1.28	
Normal	60,808	67,004	50.00%	0.00	9.70%
Worst		82,871	90.00%	1.28	

Table 55: Case Study 3 – Input parameters modified scenario approach
Source: Own representation

The next table shows all parameters for the Monte-Carlo simulations:

Asset						
Parameter	**Normal**	**Best**	**Worst**	**Mean**	**Stddev**	**Stddev**
Effort New Process	-30	-10	-40	-27	13	-48.77%
Number Tickets 2007	19,660	20,416	19,287	19,788	490	2.48%
Growth Rate Tickets	1.05	1.07	1.04	1.0533	0.0130	1.23%
Portion external FM 2007	38.00%	40.00%	36.00%	0.38	0.0156	4.11%
Percentage Rise external FM	1.50%	2.50%	0.50%	1.50%	0.0078	52.02%
Additional Degree of Automation 07	30.00%	49.00%	15.00%	31.33%	0.1379	44.00%
Percentage Rise Automation	1.25%	2.50%	0.00%	1.25%	0.0098	78.03%

Exercise Price						
Parameter	**Normal**	**Best**	**Worst**	**Mean**	**Stddev**	**Stddev**
Internal Personnel Year 1	-50.00	-35.00	-65.00	-50	12	-23.41%
Internal Personnel Year 2	-24.00	-16.80	-31.20	-24	6	-23.41%
External Personnel Year 1	-28,000	-19,600	-36,400	-28,000	6.56	-23.41%
External Personnel Year 2	-7,000	-4,900	-9,100	-7,000	1.64	-23.41%

Table 56: Case Study 3 – Input parameters Monte-Carlo simulation
Source: Own representation

In accordance with the other case studies, the calculation of the standard deviation relies on the mean and the value of the best scenario. Hence, the upside potential of the underlying asset and not the downside one enters the option valuation. Contrary to the former two case studies some difficulties arise during the Monte-Carlo simulation of the benefits' volatility.

According to Copeland and Antikarov the return of the asset is calculated using the following formula:

$$r = \ln\frac{PV_1 + C_1}{PV_0} = \ln\frac{PV_0(1+k)}{PV_0}$$

Whereas the denominator is held constant, the nominator fluctuates during the simulation. The benefits at t=1 of the normal scenario are positive and equal 19,115 EUR. A simulation model without any constraints produces negative values for $PV_1 + C_1$ in some trials, i.e. the calculation of the return is not possible. Thus, for the parameter "Portion external FM 2007" a minimum/maximum of 0%/100% is defined and for the parameter "Additional Degree of Automation 2007" a minimum/maximum of 0%/50%. With this setting no negative values occur and a volatility of the asset of 37% according to Copeland and Antikarov and of 50% according to Herath and Park is obtained. The bias in the results of the modified scenario analysis is higher than in the previous case studies because of the above constraints:

Approach	Volatility Asset	Volatility Exercise Price
Modified Scenario Analysis (Normal : Worst)	57.77%	16.58%
Modified Scenario Analysis (Best : Worst)	62.62%	22.21%
Modified Scenario Analysis (Best : Normal)	67.48%	27.83%
Simulation Herath and Park	50.24%	23.88%
Simulation Copeland and Antikarov	37.69%	16.92%
Simulation/Regression Godinho	no result	16.82%

Table 57: Case Study 3 – Volatility estimates
Source: Own representation

The simulation uses an autocorrelation coefficient of 0.9 for all variables except for the correlation between "Additional Degree of Automation 2007" and "Percentage Rise Automation". There the coefficient equals -0.9 as a rise is more difficult to achieve the higher the degree of automation already is. Unfortunately, these settings produce negative values for $PV_1 + C_1$ when using Godinho's approach, i.e. there is no outcome available. The coefficients which are used in the regression and which produce the negative values are available in the appendix. Trying to overcome this effect the formula for the return is changed as follows:

$$r = \ln\frac{\max(PV_1 + C_1;1)}{PV_0} \text{ and } r = \ln\frac{\max(PV_1 + C_1;100)}{PV_0}$$

The results of 47% and 43% show a significant upward bias in comparison to the 37% from above. Consequently, for the calculation of the option value the estimation according to Copeland and Antikarov is used instead of Godinho's. Concerning the volatility of the exercise price the calculation according to Godinho produces 17% which is quite the same outcome as the one according to Copeland and Antikarov. In general the differences between the various techniques are smaller as the volatility of the exercise price depends on a simpler model with fewer variables.

Using the Black-Scholes equation for the valuation of the option yields an outcome of 31.05 EUR. The small positive value indicates that there are cases where the option is exercised but that these cases are very seldom. This goes along with the positive/negative value of the best/normal scenario.

Parameter	Value
Underlying asset (S)	19,115
Exercise price (K)	72,062
Years to maturity (t)	1.75
Risk-free rate (rf)	3.50%
Volatility (σ)	37.69%

Option Value according to Black-Scholes: 31.05 EUR

Table 58: Case Study 3 – Option valuation according to Black-Scholes
Source: Own representation

In comparison to the negative NPV of the normal scenario, the option valuation produces clearly a higher value, however, this value is still nearly zero. The binomial tree approximation has its problems with the described characteristics. A calculation with seven periods for 1.75 years shows no exercise at the maturity date:

Input Parameters	
Risk-free rate of interest (rf)	3.56%
Discount factor	1.0088
Upward factor (u)	1.2074
Downward factor (d)	0.8282
Risk-neutral probability (p)	47.62%
Risk-neutral probability (1-p)	52.38%

Price movement of the underlying asset

Year	0	3/12	6/12	9/12	1	1 3/12	1 6/12	1 9/12
								71,492
							59,213	
						49,043		49,043
					40,619		40,619	
				33,643		33,643		33,643
			27,865		27,865		27,865	
		23,079		23,079		23,079		23,079
	19,115		19,115		19,115		19,115	
		15,832		15,832		15,832		15,832
			13,113		13,113		13,113	
				10,860		10,860		10,860
					8,995		8,995	
						7,450		7,450
							6,171	
								5,111

Option Value

Year	0	3/12	6/12	9/12	1	1 3/12	1 6/12	1 9/12
								0
							0	
						0		0
					0		0	
				0		0		0
			0		0		0	
		0		0		0		0
	0.00		0		0		0	
		0		0		0		0
			0		0		0	
				0		0		0
					0		0	
						0		0
							0	
								0

Figure 49: Case Study 3 – Binomial tree approximation Black-Scholes
Source: Own representation

An enlargement to one period per month (21 periods) provides a better approximation with 25.65 EUR, i.e. a difference of 5.4 EUR and 26%. The interested reader is referred to the appendix for details on this binomial tree. As a next step the exercise price is not held constant any more but is uncertain with a volatility of 17%. In contrast to the two case studies from above the option valuation with Margrabe's formula shows a smaller value than the result of the Black-Scholes valuation:

Parameter	Value
Asset 1: Costs (C)	60,808
Asset 2: Benefits (B)	19,115
Years to maturity (t)	1.75
Volatility Costs (σ_C)	16.92%
Volatility Benefits (σ_B)	37.69%
Correlation (ρ_{BC})	0.50

Option Value according to Margrabe: 16.59 EUR

Table 59: Case Study 3 – Option valuation according to Margrabe
Source: Own representation

The binomial tree approximation again is problematic as a tree with seven periods yields an option value of zero and one with 21 periods a result of 8.47 EUR. Both trees are available in the appendix. When taking into account a convenience yield in the option calculation it is necessary to modify the asset's PV calculation in the same manner as in the second case study. After including theoretical cash flows for the years 2004 – 2006, calculating their portion of the PV, and annualizing this result the outcome equals -10.15%. A modification of this discrete convenience yield into a continuous one leads to -10.70% per year. The subsequent figure provides the details of this calculation and the appendix shows additional information.

Year	2004	2005	2006	2007	2008	2009	...	2015
Period	**0**	**1**	**2**	**3**	**4**	**5**		**11**
Project expenses								
Personnel			-27,500	-13,200				
50 and 24 man-days á 550 EUR								
External personnel			-28,000	-7,000				
Operating expenses								
none (maintenance < 0.5 man-days per year)								
Benefits								
Reduced effort for data collection	299	1,967	2,251	2,568	2,920	3,309		6,656
Cash-flow of period t	299	1,967	2,251	2,568	2,920	3,309	0	6,656
Cash-flow of period t discounted at t = 0	299	1,785	1,854	1,919	1,980	2,037	0	2,289
Present value of the underlying asset at t = 0	**23,053**							

Sum of cash-flows which are not available for the option holder (t=0)	-3,938
Discrete convencience yield (CFs in % of PV)	-17.08%
Annualized discrete convenience yield	**-10.15%**
Continuous convencience yield (CFs in % of PV)	-18.73%
Annualized continuous convenience yield	**-10.70%**

Table 60: Case Study 3 – Calculation convenience yield
Source: Own representation

As now all necessary input parameters are available, it is possible to use Merton's formula to calculate the option value:

Parameter	Value
Underlying asset (S)	23,053
Exercise price (K)	72,062
Years to maturity (t)	1.75
Risk-free rate (rf)	3.50%
Volatility (σ)	37.69%
Convenience yield (q)	10.70%

Option Value according to Merton: 31.05 EUR

Table 61: Case Study 3 – Option valuation according to Merton
Source: Own representation

Like in the two other case studies, the result equals the option value obtained by the Black-Scholes formula. Additionally, the binomial tree approximation also provides no new insights as it produces again an outcome of 0 EUR:

Input Parameters	
Risk-free rate of interest (rf)	3.56%
Discount factor	1.0088
Upward factor (u)	1.2074
Downward factor (d)	0.8282
Risk-neutral probability (p)	47.62%
Risk-neutral probability (1-p)	52.38%
Convenience yield per year (q)	10.15%
Convenience yield per period	2.64%

Price movement of the underlying asset including the convenience yield

Year	0	3/12	6/12	9/12	1	1 3/12	1 6/12	1 9/12
Conv. yield	0.00%	2.64%	2.64%	2.64%	2.64%	2.64%	2.64%	2.64%
								73,431 71,492
							62,468 60,819	
						53,142 51,739		50,373 49,043
					45,209 44,015		42,853 41,721	
				38,459 37,444		36,455 35,493		34,555 33,643
			32,718 31,854		31,013 30,194		29,397 28,620	
		27,833 27,098		26,383 25,686		25,008 24,348		23,705 23,079
	23,053		22,444 21,851		21,274 20,713		20,166 19,633	
		19,093 18,589		18,098 17,620		17,155 16,702		16,261 15,832
			15,396 14,990		14,594 14,209		13,834 13,468	
				12,415 12,088		11,768 11,458		11,155 10,860
					10,011 9,747		9,490 9,239	
						8,073 7,860		7,652 7,450
							6,510 6,338	
								5,249 5,111

Option Value

Year	0	3/12	6/12	9/12	1	1 3/12	1 6/12	1 9/12
								0
							0	
						0		0
					0		0	
				0		0		0
			0		0		0	
		0		0		0		0
	0.00		0		0		0	
		0		0		0		0
			0		0		0	
				0		0		0
					0		0	
						0		0
							0	
								0

Figure 50: Case Study 3 – Binomial tree approximation Merton
Source: Own representation

The approach of McDonald and Siegel, which regards a convenience yield and an uncertain exercise price, exactly produces the same option value as Margrabe's formula:

Parameter	**Value**
Asset 1: Costs (C)	60,808
Asset 2: Benefits (B)	23,053
Years to maturity (t)	1.75
Volatility Costs (σ_C)	16.92%
Volatility Benefits (σ_B)	37.69%
Correlation (ρ_{BC})	0.50
Convenience yield Asset 1 (δ)	10.70%
Convenience yield Asset 2 (η)	0%

Option Value according to McDonald and Siegel: 16.59 EUR

Table 62: Case Study 3 – Option valuation according to McDonald and Siegel
Source: Own representation

The approximation using a binomial tree with seven/twenty-one periods yields 13.48/16.31 EUR. These results are nearer to the closed form outcome than before since they only show a difference of 19%/2%. The final step in the process model concerns the valuation of the American style option. The earliest point in time to start the implementation remains at 01/08/2006. Hence, no convenience yield exists up to this date to ensure that the option is still alive at t = 1.75. After this time it is necessary to calculate for each year a convenience yield. The latest date for beginning the project is at t = 9.75 (i.e. 01/08/2014) which would lead to a useful life from 01/02/2015 until 01/12/2015.

Year	**2004**	**2005**	**2006**	**2007**	**2008**	**2009**	**2010**	**2011**	**2012**	**2013**	**2014**	**2015**
Period	**0**	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**
Benefits												
Reduced effort for data collection				2,568	2,920	3,309	3,741	4,217	4,742	5,321	5,957	6,656
Cash-flow of period t	0	0	0	2,568	2,920	3,309	3,741	4,217	4,742	5,321	5,957	6,656
Cash-flow of period t discounted at t = 0	0	0	0	1,919	1,980	2,037	2,090	2,138	2,182	2,222	2,257	2,289
Present value of the underlying asset at t = 0	**19,115**											
Start of project at		t=1.75	t=2.75	t=3.75	t=4.75	t=5.75	t=6.75	t=7.75	t=8.75	t=9.75		
Lost cash-flows		periods 0,1,2	periods 0-3	periods 0-4	periods 0-5	periods 0-6	periods 0-7	periods 0-8	periods 0-9	periods 0-10		
Present value of the underlying asset before deduction of the lost cash-flow		19,115	19,115	17,195	15,215	13,178	11,088	8,950	6,768	4,547		
Sum of cash-flows which are lost, i.e. not available for the option holder (t=0)		0	1,919	1,980	2,037	2,090	2,138	2,182	2,222	2,257		
Discrete convencience yield (CFs in % of PV)		**0.00%**	**10.04%**	**11.52%**	**13.39%**	**15.86%**	**19.28%**	**24.38%**	**32.82%**	**49.65%**		

Table 63: Case Study 3 – Calculation convenience yield (American option)
Source: Own representation

The American option value with these input parameters equals 302 EUR representing an increase of circa 270 EUR in comparison to the European option value. This is explained by considering the option's exercise strategy which is shown in the next figure. As there is no early exercise at t = 1.75, it is always better to wait instead of exercising at the initial or earliest point in time. However, the major point of receiving a small option value does not change because the cases where an exercise is meaningful are very unlikely. Regarding the costs as uncertain leads again to a lower option value of 239 EUR but this value is higher than the European one of 17 EUR. The appendix contains the detailed information of all valuations.

Exercise Strategy

Year	0.00	0.25	0.50	0.75	1.00	1.25	1.50	1.75	2.00	2.25	2.50	2.75	3.00	3.25	3.50	3.75	4.00	4.25	4.50
	No	No	No	No	No	No	No	No	No	No	Early	Early	Early	Early	Early	Early	Early	Early	Early
		No	No	No	No	No	No	No	No	No	No	No	Early	Early	Early	Early	Early	Early	Early
			No	No	No	No	No	No	No	No	No	No	No	No	No	Early	Early	Early	Early
				No	No	No	No	No	No	No	No	No	No	No	No	No	No	Early	Early
					No	No	No	No	No	No	No	No	No	No	No	No	No	No	No
						No	No	No	No	No	No	No	No	No	No	No	No	No	No
							No	No	No	No	No	No	No	No	No	No	No	No	No
								No	No	No	No	No	No	No	No	No	No	No	No
									No	No	No	No	No	No	No	No	No	No	No
										No	No	No	No	No	No	No	No	No	No
											No	No	No	No	No	No	No	No	No
												No	No	No	No	No	No	No	No
													No	No	No	No	No	No	No
														No	No	No	No	No	No
															No	No	No	No	No
																No	No	No	No
																	No	No	No
																		No	No
																			No

4.75	5.00	5.25	5.50	5.75	6.00	6.25	6.50	6.75	7.00	7.25	7.50	7.75	8.00	8.25	8.50	8.75	9.00	9.25	9.50	9.75
Early	Early	Early	Early	Early	Early	Early	Early	Early	Early	Early	Early	Early	Early	Early	Early	Early	Early	Early	Early	Yes
Early	Early	Early	Early	Early	Early	Early	Early	Early	Early	Early	Early	Early	Early	Early	Early	Early	Early	Early	Early	Yes
Early	Early	Early	Early	Early	Early	Early	Early	Early	Early	Early	Early	Early	Early	Early	Early	Early	Early	Early	Early	Yes
Early	Early	Early	Early	Early	Early	Early	Early	Early	Early	Early	Early	Early	Early	Early	Early	Early	Early	Early	Early	Yes
Early	Early	Early	Early	Early	Early	Early	Early	Early	Early	Early	Early	Early	Early	Early	Early	Early	Early	Early	Early	Yes
No	No	No	Early	Early	Early	Early	Early	Early	Early	Early	Early	Early	Early	Early	Early	Early	Early	Early	Early	Yes
No	No	No	No	No	Early	Early	Early	Early	Early	Early	Early	Early	Early	Early	Early	Early	Early	Early	Early	Yes
No	No	No	No	No	No	No	No	Early	Early	Early	Early	Early	Early	Early	Early	Early	Early	Early	Early	Yes
No	No	No	No	No	No	No	No	No	No	No	Early	Early	Early	Early	Early	Early	Early	Early	Early	Yes
No	No	No	No	No	No	No	No	No	No	No	No	No	No	No	Early	Early	Early	Early	Early	Yes
No	No	No	No	No	No	No	No	No	No	No	No	No	No	No	No	No	No	No	No	No
No	No	No	No	No	No	No	No	No	No	No	No	No	No	No	No	No	No	No	No	No
No	No	No	No	No	No	No	No	No	No	No	No	No	No	No	No	No	No	No	No	No
No	No	No	No	No	No	No	No	No	No	No	No	No	No	No	No	No	No	No	No	No
No	No	No	No	No	No	No	No	No	No	No	No	No	No	No	No	No	No	No	No	No
No	No	No	No	No	No	No	No	No	No	No	No	No	No	No	No	No	No	No	No	No
No	No	No	No	No	No	No	No	No	No	No	No	No	No	No	No	No	No	No	No	No
No	No	No	No	No	No	No	No	No	No	No	No	No	No	No	No	No	No	No	No	No
No	No	No	No	No	No	No	No	No	No	No	No	No	No	No	No	No	No	No	No	No
No	No	No	No	No	No	No	No	No	No	No	No	No	No	No	No	No	No	No	No	No
	No	No	No	No	No	No	No	No	No	No	No	No	No	No	No	No	No	No	No	No
		No	No	No	No	No	No	No	No	No	No	No	No	No	No	No	No	No	No	No
			No	No	No	No	No	No	No	No	No	No	No	No	No	No	No	No	No	No
				No	No	No	No	No	No	No	No	No	No	No	No	No	No	No	No	No
					No	No	No	No	No	No	No	No	No	No	No	No	No	No	No	No
						No	No	No	No	No	No	No	No	No	No	No	No	No	No	No
							No	No	No	No	No	No	No	No	No	No	No	No	No	No
								No	No	No	No	No	No	No	No	No	No	No	No	No
									No	No	No	No	No	No	No	No	No	No	No	No
										No	No	No	No	No	No	No	No	No	No	No
											No	No	No	No	No	No	No	No	No	No
												No	No	No	No	No	No	No	No	No
													No	No	No	No	No	No	No	No
														No	No	No	No	No	No	No
															No	No	No	No	No	No
																No	No	No	No	No
																	No	No	No	No
																		No	No	No
																			No	No
																				No

Figure 51: Case Study 3 – Exercise strategy American option
Source: Own representation

Finally, the investment decision based on the option values is negative as the option values cannot compensate for the highly negative NPV of the EAI tool:

	Black-Scholes (Fixed Costs)	Margrabe (Uncertain Costs)
NPV of the IIS	-357,922	-357,922
Option Value	31	17
Strategic NPV	-357,891	-357,905
Investment decision	**NO**	**NO**

Table 64: Case Study 3 – Investment decision
Source: Own representation

5.3.3 Interpretation of the Results

Concerning the first assumption, the NPV of the IIS without consideration of the follow-on investment is negative which is in accordance with the two previous case studies. For an examination of the other assumptions the subsequent table, containing all calculated option values, is of help:

Type	Result Closed form	Binomial Tree Approximation			American Option Valuation	
		Periods per 1.75 years	Result	Differ-ence	Result	Differ-ence
Fixed Costs, No Convenience Yield	31.05	7	0.00	100.00%	301.95	872.57%
		21	37.37	20.28%		
Fixed Costs, Convenience Yield	31.05	7	0.00	100.00%		
Uncertain Costs, No Convenience Yield	16.59	7	0.00	100.00%	239.16	1341.21%
		21	8.47	48.94%		
Uncertain Costs, Convenience Yield	16.59	7	13.48	18.79%		
		21	16.31	1.74%		

Table 65: Case Study 3 – Results option valuation
Source: Own representation

Regardless of the valuation technique, the option value is slightly positive and thus clearly higher than the NPV of the normal scenario (-41,693 EUR). Hence, in most cases the option will not be exercised but there exist some cases where an exercise makes sense. This rather extreme situation has a negative impact on the accuracy of the binomial tree approximation. Even the trees with 21 periods for 1.75 years produce a bias of much more than 5%, however, in absolute figures the differences are still small. An adjustment in the valuation model for the incorporation of convenience yields provides no new insights as the results are the same. Contrary, regarding the costs as uncertain reduces the option value from 31.05 EUR to 16.59 EUR. This is quite surprising as the result should be higher than before because of the increased uncertainty. The reason for that is again the extreme situation of having nearly no cases where an exercise is suitable. However, the difference in absolute figures is once more negligible. Despite the various results the investment decision is always not to invest as the NPV of the IIS is highly negative.

The sensitivity analysis of the volatility of the benefits shows that the option value equals zero for volatilities smaller than 30%. A higher volatility leads to a higher option value and the result according to Black Scholes is always higher than Margrabe's. Moreover, the increase in the volatility also generates a high deviation in relative figures but not in absolute ones. The binomial tree approximation becomes better the higher the volatility which makes sense as then the situation becomes less extreme, i.e. more positive option values on maturity lead to a more probable exercise.

Volatility (%)	A) Black Scholes	Deviation	B) Margrabe	Deviation	BinTree BS	Difference to A)	BinTree Margrabe	Difference to B)
5	0	0%	0	0%	0	*Division by 0*	0	-100%
10	0	0%	0	0%	0	*Division by 0*	0	-100%
15	0	0%	0	0%	0	-100%	0	-100%
20	0	0%	0	0%	0	-100%	0	-100%
25	0	1%	0	1%	0	-77%	0	-95%
30	3	9%	1	7%	1	-58%	0	-65%
35	15	50%	8	46%	9	-41%	4	-45%
37.69	**31**	**100%**	**17**	**100%**	**26**	**-17%**	**8**	**-49%**
40	52	166%	29	177%	41	-21%	19	-34%
45	124	401%	79	478%	115	-7%	52	-35%
50	243	784%	169	1021%	209	-14%	140	-17%
55	413	1331%	307	1852%	404	-2%	239	-22%
60	634	2043%	496	2990%	618	-3%	433	-13%
65	904	2913%	735	4431%	826	-9%	663	-10%
70	1219	3926%	1021	6155%	1201	-1%	891	-13%
75	1573	5067%	1350	8135%	1592	1%	1170	-13%
80	1961	6318%	1716	10340%	1971	0%	1586	-8%

Table 66: Case Study 3 – Sensitivity analysis volatility
Source: Own representation

Considering the initial volatility of 37.69% and its option values of 31 EUR / 17 EUR it is obvious that the other approaches for estimating the volatility generate an upward bias in the result. They produce estimates between 50% and 70% and thus outcomes between 243 EUR / 169 EUR and 1,219 EUR / 1,021 EUR.

When changing the correlation to a value equal or less than 25% then the option value with uncertain costs is higher than the result with fixed costs. Additionally, it holds again true that an increasing option value, i.e. a decreasing correlation coefficient, improves the binomial tree approximation. All in all, the influence of varying the correlation is clearly smaller than modifications of the volatility and especially in absolute terms the estimation of 50% provides a good result:

Correlation (%)	Margrabe	Deviation	Binomial Tree 7 periods	Difference to Margrabe	Binomial Tree 21 periods	Difference to Margrabe
0	111	671.80%	74	-33.30%	83	-25.73%
5	98	588.86%	67	-31.04%	66	-32.07%
10	85	511.49%	60	-29.00%	55	-35.78%
15	73	439.80%	53	-27.39%	49	-33.14%
20	62	373.88%	46	-26.58%	43	-30.75%
25	52	313.78%	38	-27.14%	37	-28.96%
30	43	259.53%	30	-30.04%	31	-28.32%
35	35	211.12%	22	-36.88%	25	-29.79%
40	28	168.47%	14	-50.37%	18	-35.08%
45	22	131.49%	5	-75.28%	12	-47.22%
50	**17**	**100.00%**	**0**	**-100.00%**	**8**	**-48.94%**
55	12	73.76%	0	-100.00%	7	-45.17%
60	9	52.45%	0	-100.00%	5	-43.82%
65	6	35.70%	0	-100.00%	3	-49.29%
70	4	23.02%	0	-100.00%	1	-68.58%
75	2	13.88%	0	-100.00%	1	-63.53%
80	1	7.69%	0	-100.00%	0	-63.66%
85	1	3.82%	0	-100.00%	0	-84.44%
90	0	1.64%	0	-100.00%	0	-80.75%
95	0	0.58%	0	-100.00%	0	-95.71%
100	0	0.15%	0	-100.00%	0	-94.68%

Table 67: Case Study 3 – Sensitivity analysis correlation
Source: Own representation

The benefit of waiting to invest grows in absolute figures with an increasing volatility. Hence, the other techniques for estimating the volatility produce an upward bias:

	Fixed Costs			Uncertain Costs		
Volatility (%)	European Value	American Value	Difference	European Value	American Value	Difference
30	1	48	4172.55%	0	40	9565.48%
35	9	180	1896.78%	4	142	3242.12%
37.69	26	302	1077.12%	**8**	**239**	**2722.75%**
40	41	434	962.81%	19	349	1694.57%
45	115	818	608.75%	52	682	1213.82%
50	209	1,319	530.32%	140	1,134	708.83%
55	404	1,916	374.69%	239	1,693	608.42%
60	618	2,588	318.61%	433	2,341	440.22%
65	826	3,309	300.71%	663	3,046	359.63%
70	1,201	4,073	239.05%	891	3,795	325.93%
75	1,592	4,853	204.96%	1,170	4,577	291.25%
80	1,971	5,623	185.31%	1,586	5,375	238.94%

Table 68: Case Study 3 – Sensitivity analysis volatility American option
Source: Own representation

Changing the correlation coefficient again results only in minor differences and deviations between the obtained values:

Correlation (%)	Binomial Tree European	American Value	Difference	Deviation
0	74	857	1052.45%	358%
10	60	714	1084.26%	298%
20	46	578	1168.24%	242%
30	30	452	1399.37%	189%
40	14	339	2340.40%	142%
50	**0**	**239**	***Division by 0***	**100%**
60	0	155	*Division by 0*	65%
70	0	89	*Division by 0*	37%
80	0	43	*Division by 0*	18%
90	0	15	*Division by 0*	6%
100	0	3	*Division by 0*	1%

Table 69: Case Study 3 – Sensitivity analysis correlation American option
Source: Own representation

A modification of either the benefits or the costs by +/- 5% leads to a relative change between 30% and 50% in the option value. Although this is quite a high increase the modification's influence is smaller in comparison to the difference of more than 70% between calculating with fixed or uncertain costs:

	PV Benefits (t=0)	PV Costs (t=0)	Option Value Black Scholes	Option Value Margrabe	Differ-ence	Deviation Black Scholes	Deviation Margrabe
Original Values	**19,115**	**-60,808**	**31**	**17**	**87.09%**	**100.00%**	**100.00%**
Benefits +5%	20,071	-60,808	43	24	76.44%	138.69%	147.06%
Benefits -5%	18,159	-60,808	22	11	99.60%	70.28%	65.88%
Costs -5%	19,115	-63,848	23	12	98.95%	75.09%	70.62%
Costs +5%	19,115	-57,768	42	24	75.92%	133.95%	142.46%

Table 70: Case Study 3 – Sensitivity analysis present values
Source: Own representation

The sensitivity analysis for the American option provides the same insights as the option values do not change more than 25%:

	PV Benefits (t=0)	PV Costs (t=0)	American Option Value			
			Fixed Costs	Deviation	Uncertain Costs	Deviation
Original Values	**19,115**	**-60,808**	**302**	**100.00%**	**239**	**100.00%**
Benefits +5%	20,071	-60,808	363	120.09%	293	122.35%
Benefits -5%	18,159	-60,808	248	82.03%	192	80.26%
Costs -5%	19,115	-63,848	263	86.98%	204	85.19%
Costs +5%	19,115	-57,768	348	115.11%	281	117.41%

Table 71: Case Study 3 – Sensitivity analysis present values American option
Source: Own representation

Finally, the following table summarises the outcomes of the case study by investigating their effects on the assumptions:

#	Assumptions	Support (++, +, ~, –, – –)
1	Negative NPV of IIS	++
2	NPV smaller than option value	++
3	Binomial tree approximation sufficient	~
4	Model complexity – no convenience yields	++
5	Model complexity – uncertain implementation costs	– –
6	Equal investment decision – Simple model vs. complex model	++
7	Different investment decision – NPV vs. option value	– –
8	Volatility estimation according to Godinho	~
9	Rough estimation of the correlation coefficient	+
10	Minor benefit of a postponed implementation of the follow-on opportunity	+
11	Small estimation error of the PV has less influence than the model complexity	+

Table 72: Case Study 3 – Overview about the examination of the assumptions
Source: Own representation

5.4 Case Study 4: “EAI and Order Management”

5.4.1 Description of the Initial Situation

As a result of its IT strategy a subsidiary of a large-sized Austrian bank intends to implement an EAI tool. Its aim is to improve the communication between its internal and external systems:

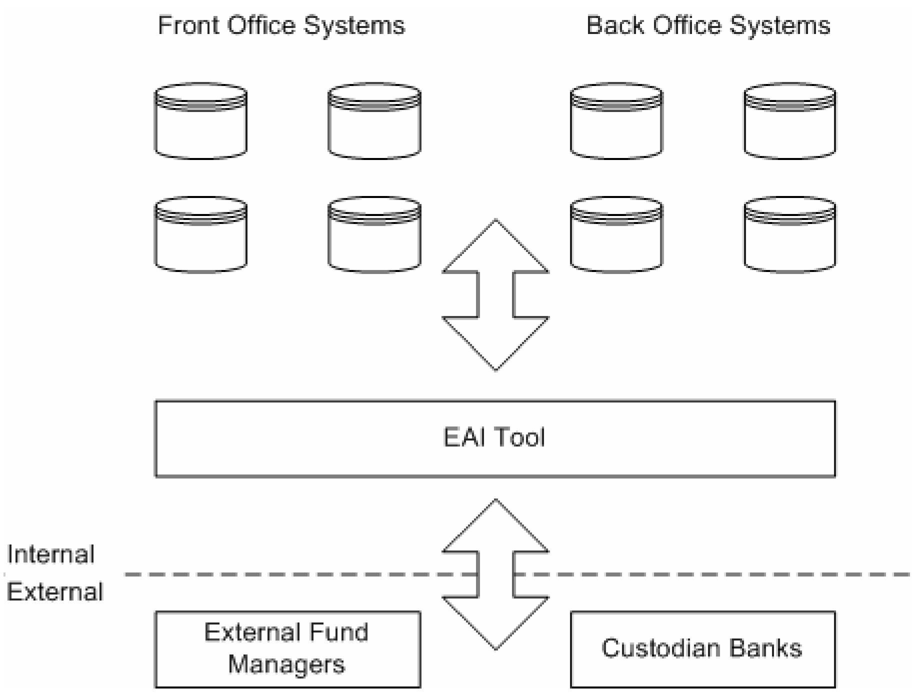

Figure 52: Case Study 4 – Data flow diagram EAI tool
Source: Own representation

The main goals are to enhance the connections between the front and back office systems, to implement a communication with external partners using the standard formats SWIFT and FIX, and to introduce a flexible basis for possible challenges in the future. Hence, the bank uses the following benefits for the calculation of the investment’s value:

- Automation of the order routing, i.e. no manual data entry
- Secure delivery of transaction data and position data, i.e. no manual checking
- Reduction in the efforts of maintaining the internal and external interfaces

The investment value is determined for three scenarios and the outcomes are: 999,319 EUR (best scenario), 514,433 EUR (normal scenario), and -510,179 EUR (worst scenario). The three calculations are available in the appendix and allow for drawing the following chart:

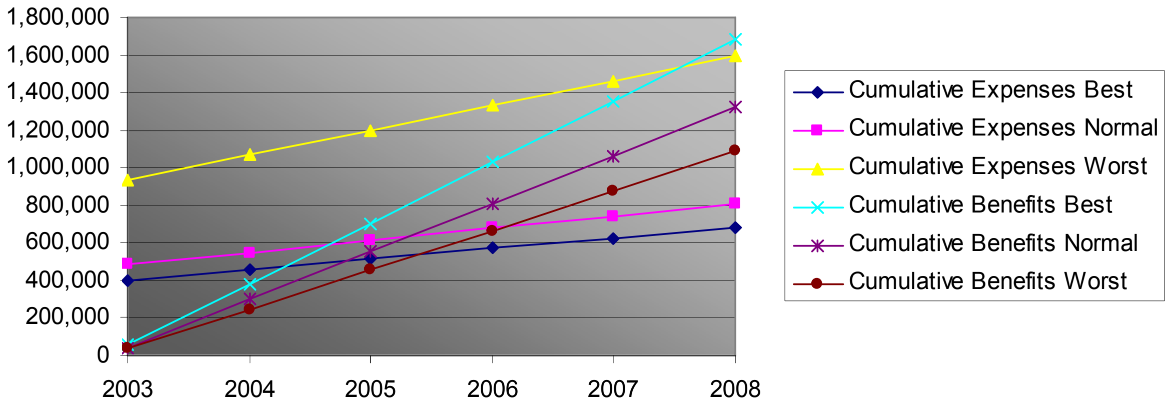

Figure 53: Case Study 4 – Cumulative expenses and benefits EAI investment
Source: Own representation

Even the cumulative benefits in the worst case are higher than the expenses in the normal or the best case. Moreover, the expenses in the worst case are offset by the benefits in the best case. All in all, the company decides to implement the EAI tool.

There are two important shortcomings in the investment valuation from above. First, there is no consideration of the time-value of money, i.e. the cash flows of the various years are not discounted but enter the calculation without any correction. Second, the automation of the order routing does not directly result from introducing the EAI system. The platform only creates the possibility for the automation and in the future there are additional expenses necessary. Thus, the need for a real option analysis arises.

5.4.2 Calculation

5.4.2.1 Phase 1: Option Identification

It has already been mentioned that the EAI tool creates the chance to automate the process of routing orders for bonds and umbrella funds. The data flow diagram of this new process looks as follows:

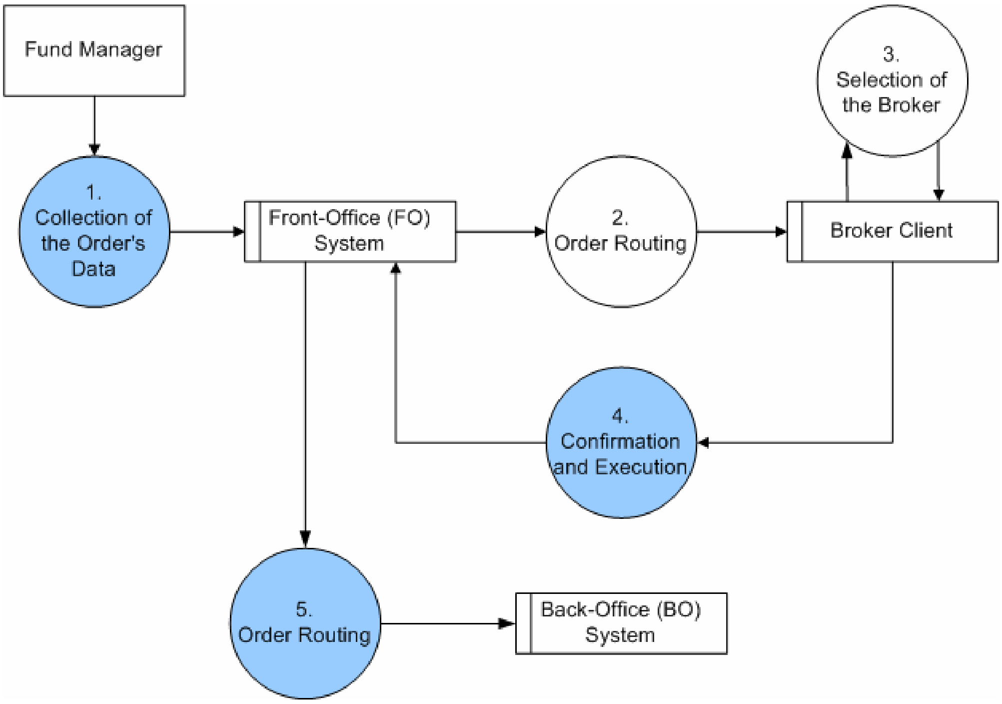

Figure 54: Case Study 4 – Data flow order routing
Source: Own representation

The fund manager creates the order and enters all relevant information in an excel sheet which is transferred to the front-office system automatically. After forwarding the order from the front-office system to the broker client and selecting the broker, the trade is confirmed and executed. The final step concerns the routing of all data to the back-office system. Hence, the new process creates the subsequent benefits:

- Reduced manual effort for collecting the order's data in the front-office system
- Reduced manual effort for reconciliation and monitoring
- Reduced manual effort for order routing
- Reduced error rate and thus reduced manual efforts
- Ex ante verification[2] from the order entry on and thus reduction of the operational risk and the potential for damage

As depicted the project "order routing" was not considered alone but as a part of the EAI project in the beginning. Later on the company created a single project with starting date 01/08/2004. Moreover, a third project was introduced with the goal to build the technical infrastructure for the communication between the EAI tool and the front-office system. The

[2] Check if the order is possible according to legal directives

goal was to connect the front-office system to the EAI tool and thus to other systems. Although this project represents a prerequisite for the project "order routing" its starting date was 15/10/2004. If the real options approach had been used, a correct treatment of these options would have led to the following project plan:

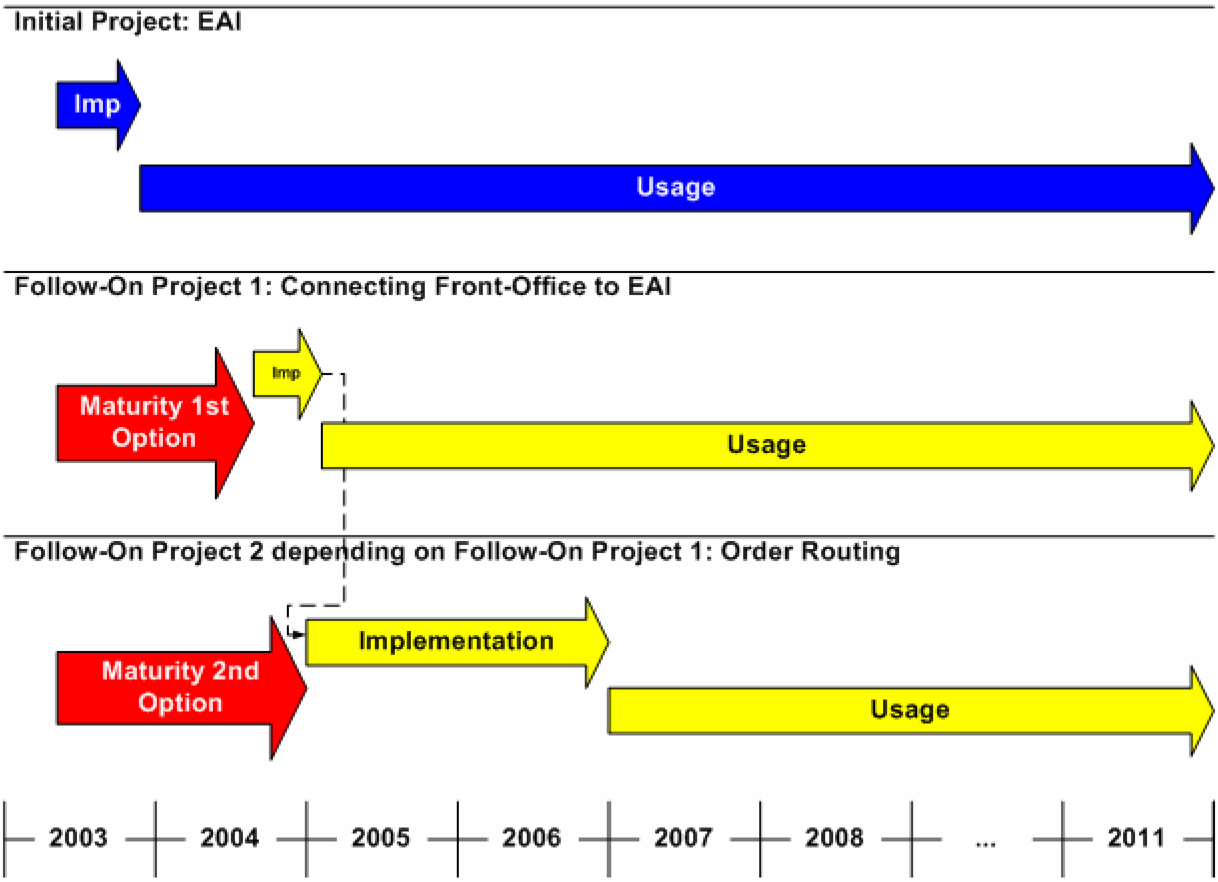

Figure 55: Case Study 4 – Timeline
Source: Own representation

Here the introduction of the EAI tool lasts from 01/05/2003 until 31/10/2003 and is followed by the implementation of follow-on project 1 (01/08/2004 – 31/01/2005). Finally, the project order routing is realised from 01/01/2005 until 31/12/2006. As the company plans a useful life of five years for the order routing all applications end at 31/12/2011.

5.4.2.2 Phase 2: NPV Calculation

Applying the NPV method corrects the initial investment valuation of the EAI tool in three areas. First, it regards the time value of money, second the cash flows of the investment order routing do not enter the calculation, and third, the useful life ends at 31/12/2011. The result is a negative NPV of -299,590 EUR. Moreover, the NPV analysis of the first follow-on investment yields -165,472 EUR. These negative values are offset by the NPV of the investment order routing which equals 1,036,115 EUR. The best scenario shows an even higher outcome of 1,874,608 EUR whereas the worst scenario results in a slightly negative value of -140,771 EUR. The sum of the NPV of the normal scenarios of both follow-on investments is 864,522 EUR resulting in an overall NPV of 564,932 EUR for all three investments. The details of all valuations are available in the appendix.

Input Parameters of the calculation					
Discount rate	5.00%	Project-Start	01.05.2003	Start of use	01.11.2003
		Project-End	31.10.2003	End of use	31.10.2008

Year	2003	2004	2005	2006	2007	2008	2009	2010	2011
Period	0	1	2	3	4	5	6	7	8
Project Expenses									
Hardware (Costs for three servers)	-36,000								
Software (License EAI Software)	-150,000								
Software (License SWIFT and FIX)	-93,653								
Internal personnel (135 man-days á 736 EUR)	-99,360								
External personnel (65 man-days á 750 EUR)	-48,750								
Operating Expenses									
Support Software	-30,000	-30,000	-30,000	-30,000	-30,000	-30,000	-30,000	-30,000	-30,000
Support Hardware	-3,600	-3,600	-3,600	-3,600	-3,600	-3,600	-3,600	-3,600	-3,600
Support SWIFT and FIX	-21,800	-21,800	-21,800	-21,800	-21,800	-21,800	-21,800	-21,800	-21,800
Internal personnel for maintenance (12 man-days per year)	-1,472	-8,832	-8,832	-8,832	-8,832	-8,832	-8,832	-8,832	-8,832
Benefits									
Savings due to a secure delivery of transaction data and position data (63 man-days)	7,728	46,368	46,368	46,368	46,368	46,368	46,368	46,368	46,368
Savings due to a reduction in the efforts of maintaining the internal and external interfaces (60 man-days)	7,360	44,160	44,160	44,160	44,160	44,160	44,160	44,160	44,160
Cash-flow of period t	-469,547	26,296	26,296	26,296	26,296	26,296	26,296	26,296	26,296
Discounted cash-flow (t = 0)	-469,547	25,044	23,851	22,715	21,634	20,604	19,622	18,688	17,798
NPV	**-299,590**								

Table 73: Case Study 4 – NPV calculation IIS
Source: Own representation

Input parameters of the calculation						
Discount rate	5.00%	Project-Start	01.08.2004		Start of use	01.02.2005
		Project-End	31.01.2005		End of use	31.12.2011

Year	**2003**	**2004**	**2005**	**2006**	**2007**	**2008**	**2009**	**2010**	**2011**
Period	**0**	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**
Project expenses									
Personnel		-64,400	-12,880						
105 man-days á 736 EUR									
Software			-54,000						
Operating expenses									
Software license			-9,000	-9,000	-9,000	-9,000	-9,000	-9,000	-9,000
Cash-flow of period t	0	-64,400	-75,880	-9,000	-9,000	-9,000	-9,000	-9,000	-9,000
Discounted cash-flow (t = 0)	0	-61,333	-68,825	-7,775	-7,404	-7,052	-6,716	-6,396	-6,092
NPV	**-171,593**								

Table 74: Case Study 4 – NPV calculation first follow-on investment
Source: Own representation

Input parameters of the calculation					
Discount rate	5.00%	Project-Start	01.01.2005	Start of use	01.01.2007
		Project-End	31.12.2006	End of use	31.12.2011

Year	2003	2004	2005	2006	2007	2008	2009	2010	2011
Period	**0**	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**
Project expenses									
Internal personnel			-282,256	-282,256					
767 man-days á 736 EUR									
External personnel			-121,000	-121,000					
Hardware				-120,000					
Operating expenses									
Hardware					-2,000	-2,000	-2,000	-2,000	-2,000
Internal personnel (2 FTE)					-353,280	-353,280	-353,280	-353,280	-353,280
Benefits									
Umbrella funds: Savings due to reduced manual efforts					86,945	99,987	114,985	132,232	152,067
Umbrella funds: Savings due to reduced error-rate					149,464	171,884	197,666	227,316	261,414
Umbrella funds: Loss Prevention through ex ante compliance					75,000	75,000	75,000	75,000	75,000
Bonds: Savings due to reduced manual efforts					91,235	100,358	110,394	121,434	133,577
Bonds: Savings due to reduced error-rate					273,705	301,075	331,183	364,301	400,731
Bonds: Loss Prevention through ex ante compliance					20,000	20,000	20,000	20,000	20,000
Cash-flow of period t	0	0	-403,256	-523,256	341,069	413,024	493,948	585,003	687,509
Discounted cash-flow (t = 0)	0	0	-365,765	-452,008	280,598	323,615	368,592	415,751	465,333
NPV	**1,036,115**								

Table 75: Case Study 4 – NPV calculation second follow-on investment
Source: Own representation

5.4.2.3 Phase 3: Option Valuation

Based on Figure 55 this case study does not value an ordinary option but instead a compound option. The EAI system creates the possibility for the connection of the front-office system which enables the project order routing. Starting with the volatility estimation according to the modified scenario approach the subsequent input parameters are necessary:

Asset					
Scenario	**PV_0**	**$PV_1 + C_1$**	**Probability**	**z-Score**	**r**
Best		2,722,308	90.00%	1.28	
Normal	1,853,889	1,946,583	50.00%	0.00	4.88%
Worst		815,544	10.00%	-1.28	

Exercise Price					
Scenario	**PV_0**	**$PV_1 + C_1$**	**Probability**	**z-Score**	**r**
Best		753,970	10.00%	-1.28	
Normal	817,773	858,662	50.00%	0.00	4.88%
Worst		963,354	90.00%	1.28	

Table 76: Case Study 4 – Input parameters modified scenario approach
Source: Own representation

The techniques using a Monte-Carlo simulation rely on the following input parameters:

Asset						
Parameter	**Normal**	**Best**	**Worst**	**Mean**	**Stddev**	**Stddev**
UF Time Savings	215	236.5	193.5	215	17	7.80%
UF Error Rate	-0.10	-0.08	-0.20	-0.13	0.04	-28.75%
UF ExAnte Savings	75,000	100,000	50,000	75,000	19,508	26.01%
UF Number of Orders 2007	15,824	17,263	15,105	16,064	935	5.82%
Bo Time Savings	150	165	135	150	11.70	7.80%
Bo Error Rate	-1.00	-0.80	-2.00	-1.27	0.36	-28.75%
Bo ExAnte Savings	20,000	30,000	10,000	20,000	7,803	39.02%
Bo Number of Orders 2007	23,800	25,964	22,719	24,161	1,407	5.82%

Exercise Price						
Parameter	**Normal**	**Best**	**Worst**	**Mean**	**Stddev**	**Stddev**
Internal Personnel Year 1	-282,256	-225,952	-338,560	-282,256	43,934	-15.57%
Internal Personnel Year 2	-282,256	-225,952	-338,560	-282,256	43,934	-15.57%
Internal Personnel Operating	-2.0	-1.8	-2.4	-2.1	0.21	-10.07%

Table 77: Case Study 4 – Input parameters Monte-Carlo simulation
Source: Own representation

The estimation of the standard deviation considers the mean and the value of the best scenario instead of the value of the worst scenario. This is consistent with the former case studies and focuses on the upside potential of the underlying. Finally, the estimation techniques produce the following outcomes:

Approach	Volatility Asset	Volatility Exercise Price
Modified Scenario Analysis (Normal : Worst)	67.88%	8.98%
Modified Scenario Analysis (Best : Worst)	47.03%	9.56%
Modified Scenario Analysis (Best : Normal)	26.17%	10.15%
Simulation Herath and Park	22.12%	13.26%
Simulation Copeland and Antikarov	15.53%	9.41%
Simulation/Regression Godinho	14.25%	9.17%

Table 78: Case Study 4 – Volatility estimates
Source: Own representation

Again the volatilities of the exercise price are quite similar but those of the asset are between 14% and 26%. Once more Godinho's result is approximated best by the approach of Copeland and Antikarov.

The valuation using Geske's formula for compound options produces 842,530 EUR which is smaller than the sum of the NPVs of 864,522 EUR. Like in previous case studies this difference is due to discounting the exercise price only with the risk free interest rate. Details about the Newton Raphson method for calculating the critical value of the underlying are available in the appendix.

Parameter	Value
Underlying option 2 (V)	1,853,889
Exercise price option 2 (M)	887,051
Years to maturity option 2 (t2)	1.67
Exercise price option 1 (K)	182,384
Years to maturity option 1 (t1)	1.25
Risk-free rate (rf)	3.50%
Volatility (σV)	14.25%

Option Value according to Geske: 842,530 EUR

Table 79: Case Study 4 – Option valuation according to Geske
Source: Own representation

As the option does not produce an increase in the investment's value it seems that its exercise is beneficiary in (nearly) all cases. This assumption is confirmed by the binomial tree approximation which can be performed either for four or 20 periods. With these settings the tree has nodes at t = 1.25 allowing for a consideration of the maturity of option one. The appendix contains both trees but as they produce the same outcomes only the shorter one is depicted here:

Input Parameters	
Risk-free rate of interest (rf)	3.56%
Discount factor	1.0147
Upward factor (u)	1.0963
Downward factor (d)	0.9121
Risk-neutral probability (p)	55.68%
Risk-neutral probability (1-p)	44.32%

Price movement of the underlying asset

Year	0	5/12	10/12	1 3/12	1 8/12
					2,678,181
				2,442,873	
			2,228,239		2,228,239
		2,032,463		2,032,463	
	1,853,889		1,853,889		1,853,889
		1,691,004		1,691,004	
			1,542,430		1,542,430
				1,406,911	
					1,283,298

Option Value

Year	0	5/12	10/12	1 3/12	1 8/12
					1,791,130
				1,386,280	
			1,186,944		1,341,188
		1,006,243		975,871	
	842,526		812,593		966,838
		664,784		634,412	
			501,135		655,380
				350,319	
					396,247

Figure 56: Case Study 4 – Binomial tree approximation Geske
Source: Own representation

For example the first node at t = 1.25 shows a value of 1,386,280 resulting from the following calculation: MAX (p * 1,791,130 + (1-p) * 1,341,188 – 182,384; 0). Furthermore, the approximation of the tree is only 4 EUR smaller than the value of the closed form solution.

The consideration of an uncertain exercise price uses its volatility of 9% and applies Carr's formula:

Parameter	Value
Asset 1: Costs (C)	817,773
Asset 2: Benefits (B)	1,853,889
Years to maturity option 2 (t2)	1.67
Exercise price option 1 (I)	171,593
Years to maturity option 1 (t1)	1.25
Volatility Costs (σ_C)	9.17%
Volatility Benefits (σ_B)	14.25%
Correlation (ρ_{BC})	0.50

Option Value according to Carr: 864,522 EUR

Table 80: Case Study 4 – Option valuation according to Carr
Source: Own representation

As this approach discounts the costs with 5% to t=0 the result of 864,522 EUR is the same compared to the NPV analysis. The binomial trees in the appendix again show that the option is exercised in each period and thus produces no increase in value. The next step is the introduction of a convenience yield. For that the calculation also accounts for the theoretical cash flows of the years 2003 – 2006. At t=0 they equal -28.39% of the PV leading to an annualized convenience yield of -18.16% and a continuous one of -20.04%.

Year	2003	2004	2005	2006	2007	2008	2009	2010	2011
Period	0	1	2	3	4	5	6	7	8
Operating expenses									
Hardware	-2,000	-2,000	-2,000	-2,000	-2,000	-2,000	-2,000	-2,000	-2,000
Internal personnel (2 FTE)	-353,280	-353,280	-353,280	-353,280	-353,280	-353,280	-353,280	-353,280	-353,280
Benefits									
Umbrella funds: Savings due to reduced manual efforts	49,711	57,168	65,743	75,604	86,945	99,987	114,985	132,232	152,067
Umbrella funds: Savings due to reduced error-rate	85,457	98,275	113,016	129,969	149,464	171,884	197,666	227,316	261,414
Umbrella funds: Loss Prevention through ex ante compliance	75,000	75,000	75,000	75,000	75,000	75,000	75,000	75,000	75,000
Bonds: Savings due to reduced manual efforts	64,970	68,546	75,401	82,941	91,235	100,358	110,394	121,434	133,577
Bonds: Savings due to reduced error-rate	194,911	205,638	226,202	248,822	273,705	301,075	331,183	364,301	400,731
Bonds: Loss Prevention through ex ante compliance	20,000	20,000	20,000	20,000	20,000	20,000	20,000	20,000	20,000
Cash-flow of period t	134,769	169,347	220,082	277,056	341,069	413,024	493,948	585,003	687,509
Cash-flow of period t discounted at t = 0	134,769	161,283	199,621	239,332	280,598	323,615	368,592	415,751	465,333
Present value of the underlying asset at t = 0	**2,588,894**								

Sum of cash-flows which are not available for the option holder	-735,005
CFs in % of PV (discrete)	-28.39%
Annualized discrete convenience yield	**-18.16%**
CFs in % of PV (continuous)	-33.39%
Annualized continuous convenience yield	**-20.04%**

Table 81: Case Study 4 – Calculation convenience yield
Source: Own representation

After the identification of the convenience yield it is possible to calculate the option value:

Parameter	Value
Underlying option 2 (V)	2,588,894
Exercise price option 2 (M)	887,051
Years to maturity option 2 (t2)	1.67
Exercise price option 1 (K)	182,384
Years to maturity option 1 (t1)	1.25
Risk-free rate (rf)	3.50%
Volatility (σV)	14.25%
Dividend (q)	20.04%

Option Value according to Geske: 842,530 EUR

Table 82: Case Study 4 – Option valuation according to Geske (convenience yield)
Source: Own representation

The result of 842,530 EUR exactly equals the outcome from above and also the binomial tree approximation produces the same value as before:

Input Parameters	
Risk-free rate of interest (rf)	3.56%
Discount factor	1.0147
Upward factor (u)	1.0963
Downward factor (d)	0.9121
Risk-neutral probability (p)	55.68%
Risk-neutral probability (1-p)	44.32%
Convenience yield per year (q)	18.16%
Convenience yield per period	8.01%

Price movement of the underlying asset

Year	0	5/12	10/12	1 3/12	1 8/12
Conv. yield	0.00%	8.01%	8.01%	8.01%	8.01%
					2,911,371 2,678,181
				2,886,796 2,655,574	
			2,862,429 2,633,159		2,422,252 2,228,239
		2,838,267 2,610,932		2,401,806 2,209,431	
	2,588,894		2,381,533 2,190,781		2,015,307 1,853,889
		2,361,431 2,172,289		1,998,296 1,838,240	
			1,981,429 1,822,724		1,676,730 1,542,430
				1,662,577 1,529,411	
					1,395,035 1,283,298

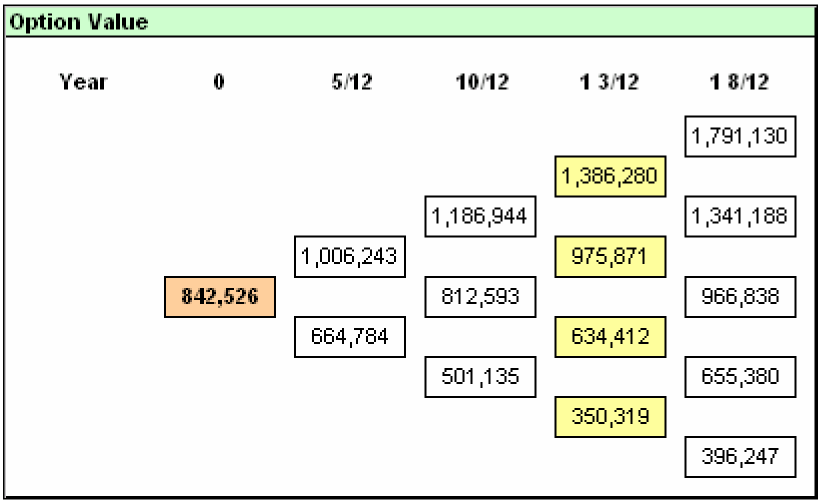

Figure 57: Case Study 4 – Binomial tree approximation Geske (convenience yield)
Source: Own representation

The appendix also contains a version of the tree with 20 periods but this one does not show any other option value. The consideration of uncertain costs in combination with convenience yields generates like before an option value of 864,522 EUR:

Parameter	Value
Asset 1: Costs (C)	817,773
Asset 2: Benefits (B)	2,588,894
Years to maturity option 2 (t2)	1.67
Exercise price option 1 (I)	171,593
Years to maturity option 1 (t1)	1.25
Volatility Costs (σ_C)	9.17%
Volatility Benefits (σ_B)	14.25%
Correlation (ρ_{BC})	0.50
Variance B/C (σ^2)	1.56%
Dividend Asset 1 (q_C)	0.00%
Dividend Asset 2 (q_B)	20.04%

Option Value according to Carr: 864,522 EUR

Table 83: Case Study 4 – Option valuation according to Carr (convenience yield)
Source: Own representation

The result of the binomial tree with only four periods equals 912,869 EUR and thus deviates by 5.59% from the closed form solution. An enlargement up to 20 periods improves the approximation showing only a difference of 1.18% and an option value of 874,718 EUR. Both trees are available in the appendix.

In order to value the option as an American one it is necessary to change the maturity of the option. The European option considers the earliest possible implementation dates, i.e. 01/08/2004 for the project front-office connection (maturity = 1.25 years) and 01/01/2005 for the project order routing (maturity 1 8/12 years). Ensuring at least a useful life of one year from 01/01/2011 until 31/12/2011 the last implementation start for the project order routing is 01/01/2009. Based on these parameters the calculation of the convenience yields is as follows:

Year	2003	2004	2005	2006	2007	2008	2009	2010	2011
Period	**0**	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**
Operating expenses									
Hardware					-2,000	-2,000	-2,000	-2,000	-2,000
Internal personnel (2 FTE)					-353,280	-353,280	-353,280	-353,280	-353,280
Benefits									
Umbrella funds: Savings due to reduced manual efforts					86,945	99,987	114,985	132,232	152,067
Umbrella funds: Savings due to reduced error-rate					149,464	171,884	197,666	227,316	261,414
Umbrella funds: Loss Prevention through ex ante compliance					75,000	75,000	75,000	75,000	75,000
Bonds: Savings due to reduced manual efforts					91,235	100,358	110,394	121,434	133,577
Bonds: Savings due to reduced error-rate					273,705	301,075	331,183	364,301	400,731
Bonds: Loss Prevention through ex ante compliance					20,000	20,000	20,000	20,000	20,000
Cash-flow of period t	0	0	0	0	341,069	413,024	493,948	585,003	687,509
Cash-flow of period t discounted at t = 0	0	0	0	0	280,598	323,615	368,592	415,751	465,333
Present value of the underlying asset at t = 0	**1,853,889**								

Start of project at	t=1.67	t=2.67	t=3.67	t=4.67	t=5.67
Lost cash-flows	periods 0-3	periods 0-4	periods 0-5	periods 0-6	periods 0-7
Present value of the underlying asset before deduction of the lost cash-flow	1,853,889	1,853,889	1,573,291	1,249,676	881,084
Sum of cash-flows which are lost, i.e. not available for the option holder (t=0)	0	280,598	323,615	368,592	415,751
Discrete convenience yield (CFs in % of PV)	**0.00%**	**15.14%**	**20.57%**	**29.49%**	**47.19%**

Table 84: Case Study 4 – Calculation convenience yield (American option)
Source: Own representation

As a next step option two, i.e. project order routing, is valued as an ordinary American option without consideration of option one. The table below shows an optimal exercise strategy of exercising as early as possible, i.e. at t = 1 8/12. This is equal to the maturity of the European option and hence, the American option value will be equal to the European one. For its calculation the maturity of option one is set at t = 1.25 to enable the early exercise at t = 1 8/12. The appendix shows that the results for fixed and uncertain costs are equal to their European counterparts.

Exercise Strategy Option 1

Year	0	2/12	4/12	6/12	8/12	10/12	1	1 2/12	1 4/12	1 6/12	1 8/12	1 10/12	2	2 2/12	2 4/12	2 6/12	2 8/12
	No	No	No	No	No	No	No	No	No	No	Early	Early	Early	Early	Early	Early	Early
		No	No	No	No	No	No	No	No	No	Early	Early	Early	Early	Early	Early	Early
			No	No	No	No	No	No	No	No	Early	Early	Early	Early	Early	Early	Early
				No	No	No	No	No	No	No	Early	Early	Early	Early	Early	Early	Early
					No	No	No	No	No	No	Early	Early	Early	Early	Early	Early	Early
						No	No	No	No	No	Early	Early	Early	Early	Early	Early	Early
							No	No	No	No	Early	Early	Early	Early	Early	Early	Early
								No	No	No	Early	Early	Early	Early	Early	Early	Early
									No	No	Early	Early	Early	Early	Early	Early	Early
										No	Early	Early	Early	Early	Early	Early	Early
											Early	Early	Early	Early	Early	Early	Early
												Early	Early	Early	Early	Early	Early
													No	No	No	Early	Early
														No	No	No	No
															No	No	No
																No	No
																	No

2 10/12	3	3 2/12	3 4/12	3 6/12	3 8/12	3 10/12	4	4 2/12	4 4/12	4 6/12	4 8/12	4 10/12	5	5 2/12	5 4/12	5 6/12	5 8/12
Early	Early	Early	Early	Early	Early	Early	Early	Early	Early	Early	Early	Early	Early	Early	Early	Early	Yes
Early	Early	Early	Early	Early	Early	Early	Early	Early	Early	Early	Early	Early	Early	Early	Early	Early	Yes
Early	Early	Early	Early	Early	Early	Early	Early	Early	Early	Early	Early	Early	Early	Early	Early	Early	Yes
Early	Early	Early	Early	Early	Early	Early	Early	Early	Early	Early	Early	Early	Early	Early	Early	Early	Yes
Early	Early	Early	Early	Early	Early	Early	Early	Early	Early	Early	Early	Early	Early	Early	Early	Early	Yes
Early	Early	Early	Early	Early	Early	Early	Early	Early	Early	Early	Early	Early	Early	Early	Early	Early	Yes
Early	Early	Early	Early	Early	Early	Early	Early	Early	Early	Early	Early	Early	Early	Early	Early	Early	Yes
Early	Early	Early	Early	Early	Early	Early	Early	Early	Early	Early	Early	Early	Early	Early	Early	Early	Yes
Early	Early	Early	Early	Early	Early	Early	Early	Early	Early	Early	Early	Early	Early	Early	Early	Early	Yes
Early	Early	Early	Early	Early	Early	Early	Early	Early	Early	Early	Early	Early	Early	Early	Early	Early	Yes
Early	Early	Early	Early	Early	Early	Early	Early	Early	Early	Early	Early	Early	Early	Early	Early	Early	Yes
Early	Early	Early	Early	Early	Early	Early	Early	Early	Early	Early	Early	Early	Early	Early	Early	No	No
Early	Early	Early	Early	Early	Early	Early	Early	Early	Early	Early	Early	Early	Early	No	No	No	No
No	No	No	No	Early	Early	Early	Early	Early	Early	Early	Early	No	No	No	No	No	No
No	No	No	No	No	No	No	No	No	No	No	No	No	No	No	No	No	No
No	No	No	No	No	No	No	No	No	No	No	No	No	No	No	No	No	No
No	No	No	No	No	No	No	No	No	No	No	No	No	No	No	No	No	No
No	No	No	No	No	No	No	No	No	No	No	No	No	No	No	No	No	No
	No	No	No	No	No	No	No	No	No	No	No	No	No	No	No	No	No
		No	No	No	No	No	No	No	No	No	No	No	No	No	No	No	No
			No	No	No	No	No	No	No	No	No	No	No	No	No	No	No
				No	No	No	No	No	No	No	No	No	No	No	No	No	No
					No	No	No	No	No	No	No	No	No	No	No	No	No
						No	No	No	No	No	No	No	No	No	No	No	No
							No	No	No	No	No	No	No	No	No	No	No
								No	No	No	No	No	No	No	No	No	No
									No	No	No	No	No	No	No	No	No
										No	No	No	No	No	No	No	No
											No	No	No	No	No	No	No
												No	No	No	No	No	No
													No	No	No	No	No
														No	No	No	No
															No	No	No
																No	No
																	No

Figure 58: Case Study 4 – Exercise strategy American option
Source: Own representation

At last, the calculations lead to a positive investment decision as the high option value offsets the negative NPV of the EAI tool:

	Geske (Fixed Costs)	Carr (Uncertain Costs)
NPV of the IIS	-299,590	-299,590
Option Value	842,530	864,522
Strategic NPV	542,939	564,931
Investment decision	☑	☑

Table 85: Case Study 4 – Investment decision
Source: Own representation

5.4.3 Interpretation of the Results

Starting with the examination of the first assumption the consequence is the same as in the previous case studies, namely a confirmation of the negative NPV of the IIS. The subsequent table summarises all results of the former calculations and offers help for investigating the other assumptions:

Type	Result Closed form	Binomial Tree Approximation			American Option Valuation	
		Periods per 1 8/12 years	Result	Difference	Result	Difference
Fixed Costs, No Convenience Yield	842,530	4	842,526	0.00%	842,526	0.00%
		20	842,527	0.00%		
Fixed Costs, Convenience Yield	842,530	4	842,526	0.00%		
		20	842,527	0.00%		
Uncertain Costs, No Convenience Yield	864,522	4	864,522	0.00%	864,522	0.00%
		20	864,522	0.00%		
Uncertain Costs, Convenience Yield	864,522	4	912,869	5.59%		
		20	874,718	1.18%		

Table 86: Case Study 4 – Results option valuation
Source: Own representation

The sum of the normal scenario NPVs of the two investments order routing (1,036,115 EUR) and front office connection (-171,593 EUR) equals 864,522 EUR. Whereas the option valuation for uncertain costs produces the same outcome, the valuation with fixed costs deviates by 2.54% which is a result of discounting the exercise price with the risk free rate. The essence is that the option produces no extra value as the investment is very good and thus, in all cases an exercise of the option is the preferred strategy. Regardless of the four types listed in the above table, the binomial tree approximation produces nearly no discrepancy in the outcomes.

Changing the volatility while holding all other parameters constant shows that the option value does hardly change until a volatility of 45%/50% is reached. Thus, most of the approaches for estimating the volatility result in the same option value. The binomial tree approximates the outcome of the closed form solution very well, regardless of the volatility. When the volatility reaches a high level the option value with fixed costs is larger than the one with uncertain costs.

Volatility (%)	A) Geske	Deviation	B) Carr	Deviation	BinTree Geske 20	Difference to A)	BinTree Carr 20	Difference to B)
5	842,526	100%	864,522	100%	842,526	0%	864,522	0%
10	842,526	100%	864,522	100%	842,526	0%	864,522	0%
14.25	**842,530**	**100%**	**864,522**	**100%**	**842,527**	**0%**	**864,522**	**0%**
15	842,535	100%	864,521	100%	842,527	0%	864,522	0%
20	842,860	100%	864,431	100%	842,788	0%	864,542	0%
25	844,745	100%	864,044	100%	844,566	0%	864,982	0%
30	849,582	101%	864,208	100%	849,176	0%	867,060	0%
35	857,951	102%	866,804	100%	858,191	0%	871,418	1%
40	869,735	103%	873,141	101%	868,445	0%	880,173	1%
45	884,499	105%	883,479	102%	883,912	0%	891,768	1%
50	901,730	107%	897,439	104%	903,135	0%	903,336	1%
55	920,945	109%	914,406	106%	922,505	0%	921,850	1%
60	941,726	112%	933,760	108%	941,933	0%	942,305	1%
65	963,722	114%	954,954	110%	962,571	0%	962,575	1%
70	986,645	117%	977,534	113%	987,108	0%	982,649	1%

Table 87: Case Study 4 – Sensitivity analysis volatility
Source: Own representation

In accordance with the results of the previous case studies the correlation coefficient has a minor impact on the option value:

Correlation (%)	Carr	Deviation	Binomial Tree 4 periods	Difference to Carr	Binomial Tree 20 periods	Difference to Carr
0	864,450	99.99%	864,522	0.01%	864,532	0.01%
10	864,480	100.00%	864,522	0.00%	864,526	0.01%
20	864,501	100.00%	864,522	0.00%	864,524	0.00%
30	864,513	100.00%	864,522	0.00%	864,522	0.00%
40	864,519	100.00%	864,522	0.00%	864,522	0.00%
50	**864,522**	**100.00%**	**864,522**	**0.00%**	**864,522**	**0.00%**
60	864,522	100.00%	864,522	0.00%	864,522	0.00%
70	864,522	100.00%	864,522	0.00%	864,522	0.00%
80	864,522	100.00%	864,522	0.00%	864,522	0.00%
90	864,522	100.00%	864,522	0.00%	864,522	0.00%
100	864,522	100.00%	864,522	0.00%	864,522	0.00%

Table 88: Case Study 4 – Sensitivity analysis correlation
Source: Own representation

Furthermore, the volatility parameter has a small influence on the difference between the European and the American option value. Hence, the benefit of waiting to invest remains small, even with higher volatility.

	Fixed Costs			Uncertain Costs		
Volatility (%)	European Value	American Value	Difference	European Value	American Value	Difference
5	842,526	842,526	0.00%	864,522	864,522	0.00%
10	842,526	842,526	0.00%	864,522	864,522	0.00%
14.25	**842,527**	**842,526**	**0.00%**	**864,522**	**864,522**	**0.00%**
15	842,527	842,526	0.00%	864,522	864,522	0.00%
20	842,788	842,579	-0.02%	864,542	864,526	0.00%
25	844,566	844,140	-0.05%	864,982	864,866	-0.01%
30	849,176	848,812	-0.04%	867,060	867,595	0.06%
35	858,191	859,845	0.19%	871,418	872,934	0.17%
40	868,445	875,193	0.78%	880,173	884,323	0.47%
45	883,912	892,449	0.97%	891,768	900,117	0.94%
50	903,135	914,873	1.30%	903,336	921,690	2.03%
55	922,505	939,614	1.85%	921,850	944,872	2.50%
60	941,933	966,691	2.63%	942,305	970,151	2.96%
65	962,571	994,150	3.28%	962,575	995,634	3.43%
70	987,108	1,021,538	3.49%	982,649	1,025,454	4.36%
75	1,011,453	1,049,736	3.78%	1,002,519	1,056,016	5.34%
80	1,035,563	1,081,410	4.43%	1,027,464	1,086,520	5.75%

Table 89: Case Study 4 – Sensitivity analysis volatility American option
Source: Own representation

Also the impact of the correlation on the American option value is negligible:

Correlation (%)	Binomial Tree European	American Value	Difference	Deviation
0	864,522	864,522	0.00%	100%
10	864,522	864,522	0.00%	100%
20	864,522	864,522	0.00%	100%
30	864,522	864,522	0.00%	100%
40	864,522	864,522	0.00%	100%
50	**864,522**	**864,522**	**0.00%**	**100%**
60	864,522	864,522	0.00%	100%
70	864,522	864,522	0.00%	100%
80	864,522	864,522	0.00%	100%
90	864,522	864,522	0.00%	100%
100	864,522	864,522	0.00%	100%

Table 90: Case Study 4 – Sensitivity analysis correlation American option
Source: Own representation

A modification of the present values of the benefits (the costs) by +/-5% leads to a change in the option value of +/-10% (+/-5%). Hence, this small alteration influences the result more than regarding the costs as fixed or uncertain.

	PV Benefits (t=0)	PV Costs (t=0)	Option Value Geske	Option Value Carr	Differ-ence	Deviation Geske	Deviation Carr
Original Values	**1,853,889**	**-817,773**	**842,530**	**864,522**	**-2.54%**	**100.00%**	**100.00%**
Benefits +5%	1,946,583	-817,773	935,221	957,217	-2.30%	111.00%	110.72%
Benefits -5%	1,761,194	-817,773	749,845	771,824	-2.85%	89.00%	89.28%
Costs -5%	1,853,889	-858,662	800,697	823,631	-2.78%	95.03%	95.27%
Costs +5%	1,853,889	-776,885	884,366	905,411	-2.32%	104.97%	104.73%

Table 91: Case Study 4 – Sensitivity analysis present values
Source: Own representation

This insight holds also true for the American style option:

	PV Benefits (t=0)	PV Costs (t=0)	American Option Value			
			Fixed Costs	Deviation	Uncertain Costs	Deviation
Original Values	**1,853,889**	**-817,773**	**842,526**	**100.00%**	**864,522**	**100.00%**
Benefits +5%	1,946,583	-817,773	935,220	111.00%	957,217	110.72%
Benefits -5%	1,761,194	-817,773	749,831	89.00%	771,828	89.28%
Costs -5%	1,853,889	-858,662	800,687	95.03%	823,634	95.27%
Costs +5%	1,853,889	-776,885	884,365	104.97%	905,411	104.73%

Table 92: Case Study 4 – Sensitivity analysis present values American option
Source: Own representation

Finally, the subsequent table outlines the findings of this case study and their effect on the assumptions to be investigated:

#	Assumptions	Support (++, +, ~, –, – –)
1	Negative NPV of IIS	++
2	NPV smaller than option value	––
3	Binomial tree approximation sufficient	++
4	Model complexity – no convenience yields	++
5	Model complexity – uncertain implementation costs	––
6	Equal investment decision – Simple model vs. complex model	++
7	Different investment decision – NPV vs. option value	––
8	Volatility estimation according to Godinho	––
9	Rough estimation of the correlation coefficient	++
10	Minor benefit of a postponed implementation of the follow-on opportunity	++
11	Small estimation error of the PV has less influence than the model complexity	––

Table 93: Case Study 4 – Overview about the examination of the assumptions
Source: Own representation

5.5 Cross Case Analysis

The aim of this section is to consolidate the results of the option valuations of each case study in order to make a final judgement on the assumptions. All cases show quite the same maturity and thus use the same risk free interest rate for the calculations. The cases number two and four also have nearly the same volatility and reveal almost a positive NPV in all three scenarios. Contrary, case three displays practically a negative NPV in all three scenarios although having a high volatility. Moreover, the NPVs of case one diverge from one another with different signs but lead to the lowest volatility of all four cases. The following figure depicts the described setting of the input parameters and illustrates the differences between the various outcomes of the option calculations. For this classification the figure uses a scale with three different attributes, namely high, middle, and low. High/middle/low refers to a bias of >=10% / between 5% and 10% / <= 5% from the European option value obtained by a closed form solution with uncertain costs, no dividends, and a volatility estimation according to Godinho.

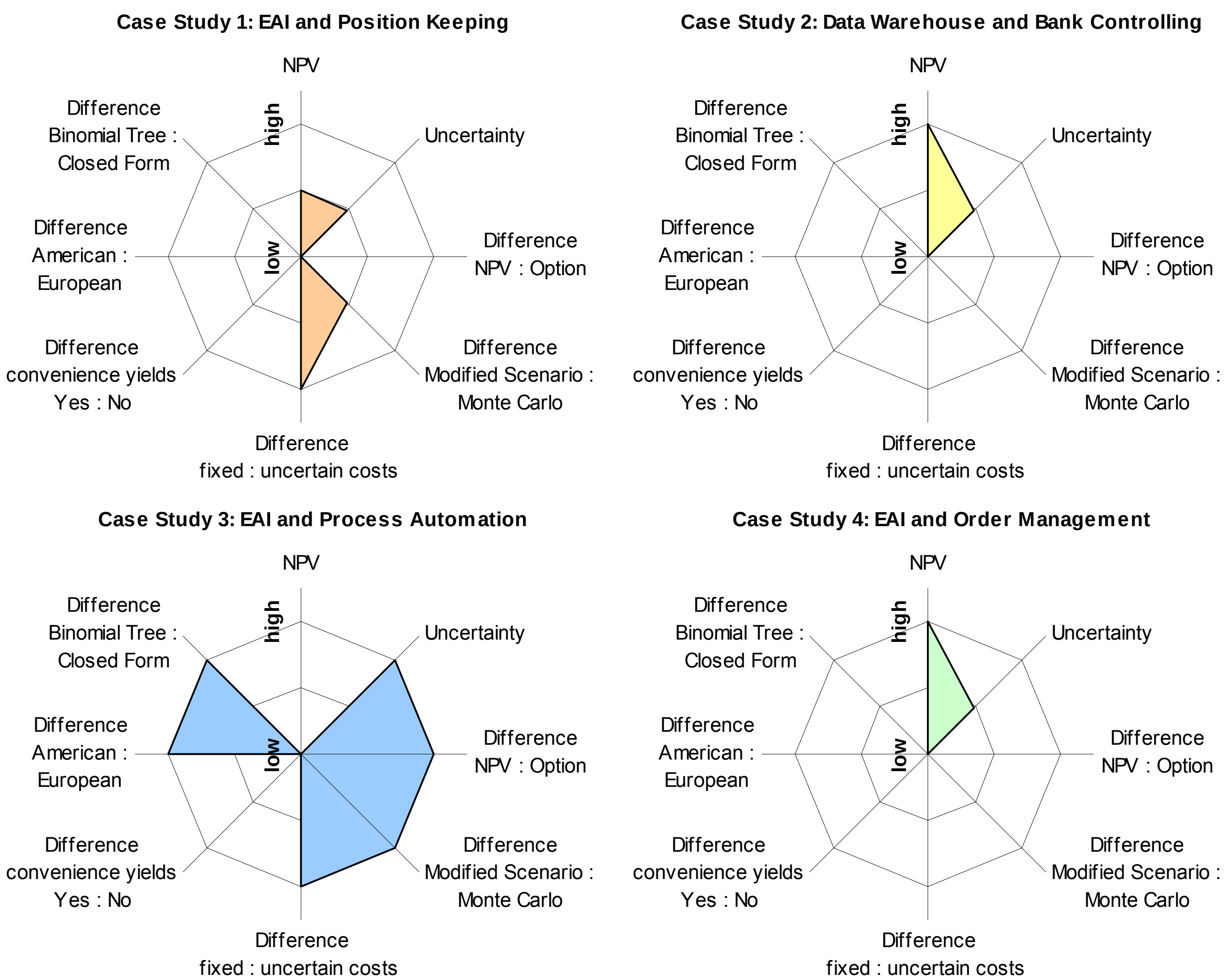

Figure 59: Cross case analysis results
Source: Own representation

To summarise, case two and four are not only equal concerning their input parameters but also in their results. The situation of the first case is quite similar to them and case three shows big differences but only in relative terms and not in absolute ones. Beside case study three there is only a minor discrepancy between the NPV of the normal scenario and the option value. Hence, the final process model has to account for the fact that a simple NPV analysis is sufficient for the investment decision in particular cases. The more different the NPV scenarios and the higher the uncertainty the need for an option analysis rises. Especially when concentrating not only on the investment decision but also on the management of the options in the future then it is worth to perform a ROA. A comparison between the option value obtained by a rough volatility estimation via the modified scenario approach (best : normal scenario) and the value obtained by Godinho's Monte-Carlo approach shows only a minor discrepancy for the cases two, four, and even one. The deviation of the option value with fixed costs and the outcome with uncertain costs is partly due to the already mentioned discounting problem. On the one hand, it is possible to modify the Black-Scholes equation in order to discount the exercise price not with the risk free interest rate. On the other hand, it is not a big effort to calculate the option's value with uncertain costs, especially when using the modified scenario approach. Moreover, a proper cash flow model eliminates the need to account for convenience yields and thus again simplifies the valuation. In each case study the surplus value of the American option is limited as early exercise is mostly the best strategy. However, the binomial tree approximates the closed form outcomes sufficiently and thus it is easy to perform the American option valuation. Furthermore, the American valuation improves the quality of managing the options in the future until their maturity date. An insight gained by the application of the Black-Scholes formula is that the strike price should be adjusted not with the risk free rate of interest but with one adjusted for risk.

After investigating the outcomes of Figure 59 and combining them with the previous results the analysis of the assumptions leads to the following:

#	Assumptions	Support (++, +, ~, -, - -)				
		Case 1	Case 2	Case 3	Case 4	Overall
1	Negative NPV of IIS	++	++	++	++	++
2	NPV smaller than option value	--	--	++	--	-
3	Binomial tree approximation sufficient	++	++	~	++	++
4	Model complexity – no convenience yields	++	++	++	++	++
5	Model complexity – uncertain implementation costs	++	--	--	--	~
6	Equal investment decision – Simple model vs. complex model	++	++	++	++	++

#	Assumptions	Support (++, +, ~, -, - -)				
		Case 1	Case 2	Case 3	Case 4	Overall
7	Different investment decision – NPV vs. option value	--	--	--	--	**--**
8	Volatility estimation according to Godinho	-	--	~	--	**-**
9	Rough estimation of the correlation coefficient	++	++	+	++	**++**
10	Minor benefit of a postponed implementation of the follow-on opportunity	++	++	+	++	**++**
11	Small estimation error of the PV has less influence than the model complexity	--	--	+	--	**--**

Table 94: Cross case analysis – examination of the assumptions
Source: Own representation

Whereas the NPV of the IIS without follow-on opportunities is, as expected, negative, the option value does not exceed the NPV of the follow-on investment in all cases. The binomial tree produces very good approximation as even in case three the deviations are small in absolute numbers. There is no need to use a more complex model which accounts for convenience yields and also the consideration of uncertain implementation costs is not totally necessary. The investment decisions obtained by the different methods are all the same because they are a clear yes or a clear no. Unfortunately, none of the four cases showed a situation where the strategic NPV is around zero. Not all volatility estimation techniques other than Godinho’s produce a significant bias in the option value. Especially the approach of Copeland and Antikarov generates nearly the same outcomes in the case studies. Moreover, the modified scenario approach (best : normal scenario) produces a good approximation with regard to its low effort. However, as there were no cases with a high volatility, a clearly positive NPV in the best scenario, and a clearly negative NPV in the worst scenario Godinho’s approach could not prove its superiority. Furthermore, the method is not applicable on case three. The various sensitivity analyses of the correlation coefficient demonstrate that its influence on the option value is only minor. Also not observable is a significant influence of an American style calculation on the option value. Finally, the sensitivity analysis of the present values indicates their importance, i.e. their estimation is a very crucial task of the option valuation.

All in all the benefit of the option valuation compared to the NPV is smaller than expected which might be an outcome of the following issues:

1. underestimation of the volatility as the calculation was carried out ex post instead of ex ante

2. application of a traditional useful life of the IIS instead of a longer time period which accounts for the optional follow-on investments and increases especially the American option value
3. untrained option thinking in the company, i.e. some options are not identified which would be very uncertain and thus especially valuable
4. the real options approach generates benefits also after the investment decision by managing the options in the future

The first point addresses the fact that an estimation of the uncertainty of cash flows after the project implementation probably leads to a downward bias. Second, the determination of the useful life of the IIS has to regard also the possible follow-on investments, i.e. it should be higher than in an ordinary investment decision where considering the IIS alone. Third, it is necessary to train and practice the process of finding options in order to capture them all. Here it is critical to drum the way of option thinking into the people’s heads and thus to ensure that also formerly unknown opportunities enter the calculation. Fortunately, in practice the efforts will be boosted if the investment decision depends on the identification of enough options. At last, nobody should forget the benefits of ROA which come into play after the investment decision stemming from the management of the options until their maturity date. In this context the term management refers not only to existing option but also to the discovery of new options.

5.6 Simplified Process Model

5.6.1 Creation of the Model

Finally, the empirical analysis concludes with the conversion of the comprehensive process model (see 4.2.2) into the simplified process model. The most important insight of the case studies is that an option valuation can generate an additional value but only in certain cases. Moreover, it is essential to spend enough time on the calculation of the NPVs as well as for the identification of all options. This is in accordance with de Jong et al. [DeRV99] who point out that one danger in conducting a ROA lies in concentrating on the model instead of the adequate determination of the inputs.

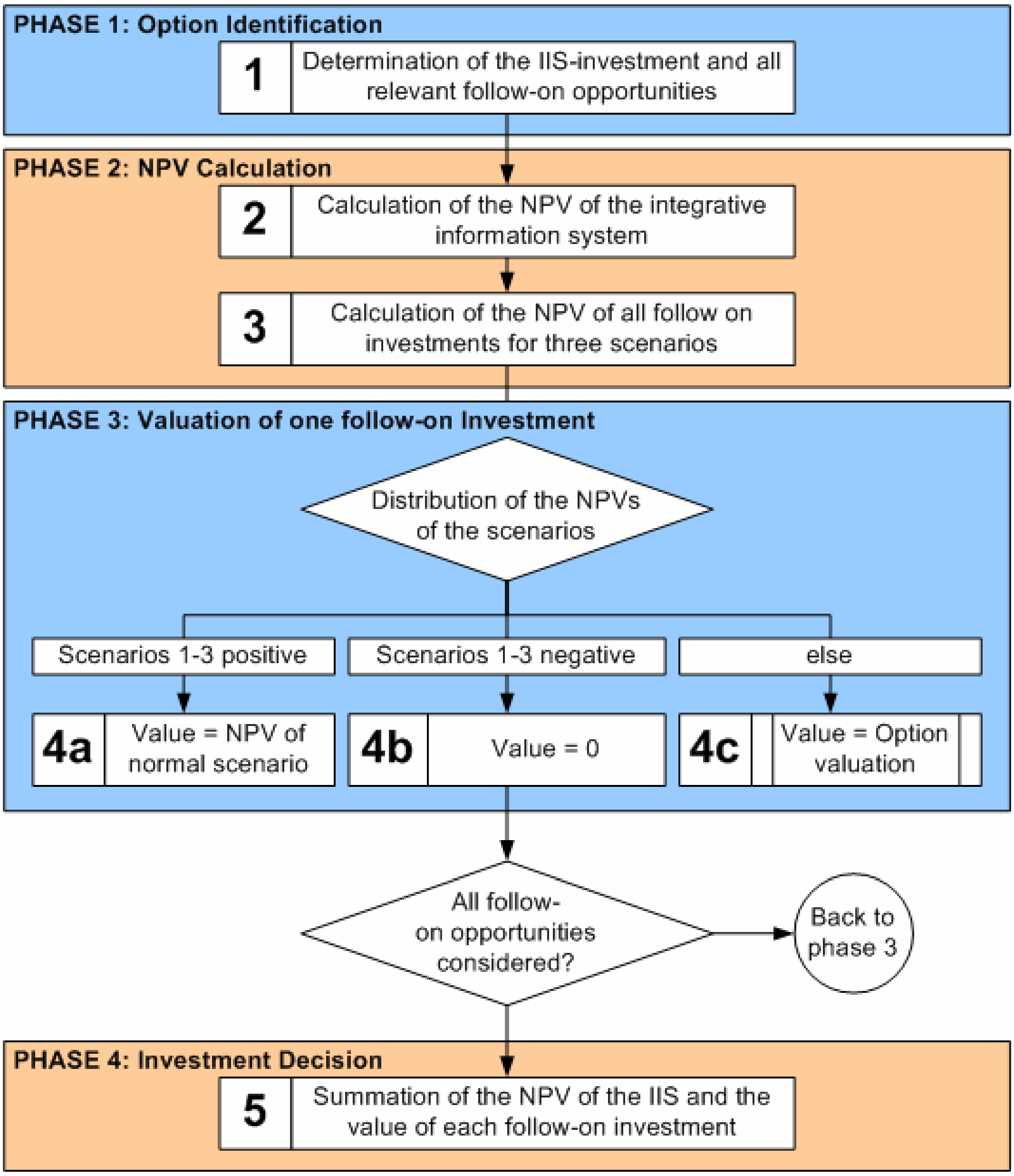

Figure 60: Simplified process model
Source: Own representation

Whereas the phases one, two, and four have not changed in comparison to the comprehensive process model, phase three is modified. Depending on the follow-on investment's NPVs of the scenarios best/normal/worst a decision is made whether or not an option valuation makes sense. When all three NPVs are positive then the option is exercised at a very high probability and thus does not produce any extra value. Contrary, if all NPVs are negative then the option is not exercised at a very high probability and thus leads to a value of zero. If the worst NPV is nearly positive or the best NPV nearly negative then the experienced decision-maker will also perform step 4a or 4b instead of 4c. However, if the signs of the NPVs are different, i.e.

higher uncertainty is present and the investment can be positive as well as negative, then it is wise to carry out a ROA:

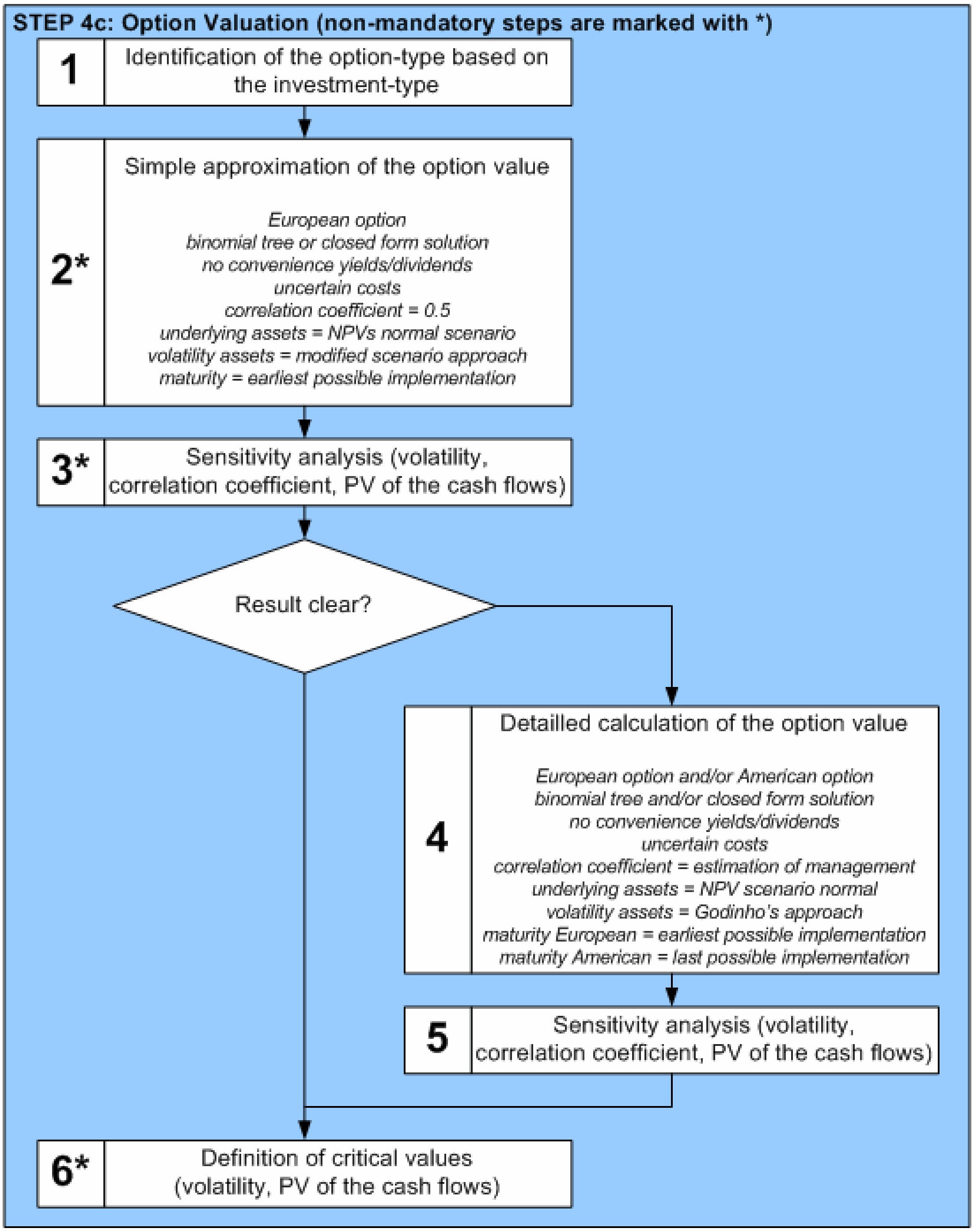

Figure 61: Simplified process model – option valuation
Source: Own representation

The steps of identifying the option, calculating its value, and conducting a sensitivity analysis are already well known (for a description see 4.2.2 Comprehensive Process Model). New in the model is the possibility to approximate the result with an easy and quick valuation. There, an European option value is computed with no convenience yields, uncertain costs, a correlation coefficient of 0.5, and a maturity equal to the earliest possible implementation date. The underlying assets (benefits and costs) are observable in the PV calculation of the normal scenario and their volatility is a result of an estimation according to the modified scenario approach (best : normal). The application of a closed form solution or a binomial tree approximation depends on the preference of the individual person. Whereas it is easy to automate a closed form solution a binomial tree representation is much more understandable. This might be of great help if e.g. the top-management has to be convinced of the results of an ROA. Furthermore, the closed form solutions for compound options are quite complicated to implement because of the bivariate normal distribution and the Newton Raphson method. When building a binomial tree the question arises how many periods the tree should contain. The answer depends mostly on the individual case and its time to maturity but a number of 15 to 20 seems to be appropriate. A sensitivity analysis of the parameters volatility, PV benefits, PV costs, and correlation coefficient shows whether or not the outcome is robust and clear. If this does not hold true then a more detailed ROA is necessary. There it is possible to account for an improved estimation of the volatility using Godinho's approach and to consider an American option. After the valuation there is again a sensitivity analysis performed to check the robustness of the outcomes. The final step of defining critical values represents a new one. It uses the results of the sensitivity analyses and determines e.g. for the volatility a lower limit. For numbers above this frontier the option value is at least still positive or contributes significantly to the compensation of the negative NPV of the IIS. These critical values are of help when managing the options in the future. Another important information for the future management might be the probability of exercising the option. This allows not only to say how much the follow-on opportunity is worth but also how high the probability of its implementation is. However, this last step and the steps two and three are not mandatory but recommended for performing a ROA.

To summarise, the simplified process model relies on the following assumptions which are an outcome of the case studies investigated:

- An option valuation produces no benefits compared to the NPV method if the scenario analysis shows (nearly) only positive or negative NPVs in all scenarios. In other situations a ROA shows its strengths.

- Crucial tasks are to identify the follow-on opportunities and to estimate the underlying asset using a PV analysis. Thus, it is more important to spend enough time on these steps than to accomplish a highly sophisticated ROA.
- It is worth to account for uncertain costs in the option valuation.
- There is no need to consider for convenience yields/dividends if the cash flow model is set up properly.
- A simple volatility estimation using the modified scenario approach provides a good first guess but for exact valuations a Monte-Carlo approach is necessary.
- The correlation coefficient has only a minor influence on the option value compared to the other input parameters like the volatility or the assets' value.
- The binomial tree method approximates the results of a closed form solution sufficiently.
- Due to the limited useful life of the IIS which represents a prerequisite of the follow-on investments an American option valuation produces hardly any surplus value. This can be overcome by calculating with a longer useful life of the IIS.

5.6.2 Discussion of the Model

A final discussion of the simplified process model and its applicability in practice with the partners of the case studies revealed the following issues[3]. The representative of the fourth case sees the major benefit of the model especially in thinking about possible follow-on opportunities. Usually the company does not try to identify follow-on projects and thus this would bring a considerable improvement in the investment decisions. In his opinion, the main problem is to gather all relevant data, even for the NPV analysis and its scenarios. The numbers do not exist and their estimation is very difficult. Once these numbers are gathered, the actual performance of an ROA is no big problem as it is possible to train the employees in this area.

Quite a similar point of view has the partner of the third case study who also sees the advantages of identifying follow-on opportunities and thinking in scenarios. According to him, the company already considers follow-on projects in some investment decisions intuitively but has no clear process model how to deal with them. Accounting for uncertainty of future events by calculating the NPV for three scenarios would lead to a further

[3] The model was only discussed with the contact persons of the cases two, three, and four as the data of case one was not collected directly but origins from another research (see 4.3.2).

improvement of the current situation of the company. At the moment, the uncertainty often leads to an upward bias in the NPV calculation as the involved people are convinced of "their" project, focus more on the best than on the normal case, and in general try to justify the investment. By computing three NPVs, these people have the chance to estimate a really good scenario and thus are not forced to push the normal scenario. The actual option calculation of step 4c appears attractive since it consolidates the scenarios and produces one number which is very striking in presentations for the top-management. Nevertheless, the general attitude towards the option calculation is a little bit reluctant due to its complexity. Moreover, its application is only for really big IT investments conceivable, like the implementation of an IIS.

The contact person of the second case imposes another view as he is an employee of a consultant company. His major interest is in using the model when calculating a price quotation of their services and products. Here the model can be used to justify high costs of the investment in the IIS as well as to sell additional services which use the IIS. One of the consultant's common problems when creating an offer is to estimate the costs and benefits of the customer. As the consulting company has not all relevant data of their customers it has to use data from the past to estimate these numbers. However, the actual option calculation does not stoke any fears but most of the follow-on projects are expected to be highly positive. In such a case it is not necessary to conduct a ROA but sufficient to use the NPV of the investment. Hence, again the main improvement is to start thinking about existing follow-on opportunities. The calculation of their value and the options created by them is also important but comes only second.

All in all, it is already an important renewal for the companies to think of follow-on projects and of different scenarios. Hence, the process model offered by this thesis provides valuable assistance. Once the decision-makers and analysts are used to these issues they will be more interested in an accurate determination of their alternatives and opportunities. At that time the real options method will show its strengths and thus increase its importance.

Part 6: CONCLUSION

6.1 Summary and Conclusion

A major challenge for IT research lies in the valuation of IT infrastructure investments and, in particular, in the valuation of integrative information systems (IIS). The need for integrating systems stems from many reasons as for example the high number of mergers and acquisitions in the last years. As projects for the implementation of IIS require high expenditures a well-founded investment decision is indispensable. Unfortunately, traditional budgeting methods like the NPV-method undervalue an IIS by not accounting for follow-on opportunities in an adequate way. This is even more important because an IIS does not generate much direct benefit but offers the possibility to implement follow-on investments that use the platform and create the positive cash flows. Previous research has already demonstrated the superiority of the real options analysis (ROA) in such situations but in practice ROA suffers from a lack of appliance mainly due to its complexity.

This thesis exactly bridges this gap and develops a process model for the valuation of investments in IIS. It starts with an explanation of pricing models for financial options which are the basis for an assessment with the real options approach. Here, the subjects of investigation are so-called "growth options" where the IIS entitles the management to build follow-on projects using the integrated data and functions of the IIS. Hence, the decision-makers hold options on the follow-on investments and have some time to consider whether or not to exercise them. This managerial flexibility and the additional time to discover more information about the future is worthwhile and increases the overall value of the IIS. Moreover, this study explains the estimation of all ROA-input parameters in detail. Especially the determination of the volatility, i.e. the uncertainty of the asset's growth rate, is considered in depth including straightforward methods as well as sophisticated approaches conducting Monte-Carlo simulations. After a review of previous research a comprehensive process model is developed which contains simple and complex valuation models. Next, the thesis applies this comprehensive model on four case studies and thus explains how a decision-maker can perform a ROA in practice. As a result of the outcomes of the cases, it is possible to develop the desired simple process model.

The insights of the empirical analyses are as follows: first, the most important issue is to spend enough time on identifying potential follow-on opportunities. In other words, to regard all possible sources of value is more essential than how they are actually priced. Second, the case studies show that a ROA has its strengths but that there are situations where it is sufficient to perform a NPV analysis only. This is the case when the follow-on investment is either very good or very bad and thus the option is exercised for sure or not at all. Hence, the simple process model contains a point where it considers three NPV scenarios (best/normal/worst) and decides whether or not to perform a ROA. Third, it is observable that the influence of the complexity of the valuation model on the result is lower than the influence of the most important input factor, namely the value of the underlying asset. Consequently, the efforts should be concentrated on this issue instead of highly sophisticated valuation models. The simple process model accounts for this fact by recommending a straightforward option valuation in the first place. An elaborate ROA is only performed when a sensitivity analysis shows that the results of the first valuation are not robust.

More precisely, the straightforward option valuation uses a model which considers a fixed point in time for starting the follow-on project. In most cases this is sufficient as a postponement does not create much benefit due to the limited useful life of the IIS. Moreover, it regards the implementation costs as uncertain and uses either a binomial tree approximation or a closed form solution. The case studies demonstrate that mostly the first one is precise enough and easier to understand for the management. By modelling the underlying cash flows in an adequate way there is no need to account for convenience yields. Finally, the volatility parameter is estimated by an easy approach which uses the different NPV scenarios best/normal/worst. The elaborate ROA allows for a consideration of a variable point of implementation and uses quite a complicated method for the assessment of the volatility. This method relies on a Monte-Carlo simulation of the investment's cash flows and a regression of the investment's behaviour after the option maturity.

A final discussion of the developed process model with the partners of the case studies revealed that they see the model's benefits not only represented by the number produced but in addition by the insights gained about the follow-on opportunities and the consideration of uncertainty via scenarios. Moreover, the actual conduction of a ROA does not seem to be frightening any more and especially the straightforward option valuation is regarded as applicable in practice.

6.2 Future Research

The most promising issue for the future is to apply the simplified process model to additional real cases. This would lead to more information on the follow-on investments and on the question whether or not they are clearly positive or negative. When being clearly positive or negative the thesis showed that a NPV analysis is sufficient as a ROA does not generate any surplus value. For cases where a ROA makes sense, additional case studies would provide further insights concerning the accuracy of the straightforward option valuation in comparison to the detailed one. Another interesting area for future research represents the determination of an appropriate discount rate and a convenient useful life of the IIS. In the case studies both of them seemed to be more a guess than a well-founded estimate which is quite dangerous as they influence the investment's value to some extent. Whereas the impact of the former one is clear, the latter one's is especially interesting in the context of American options, i.e. when postponing the start of the follow-on projects.

Beside these rather practical topics academic research can address the influence of the following issues in order to improve the accuracy of the (detailed) option valuation:

- more than one source of uncertainty
- relationships between options
- nested options

First, all models of this thesis only account for one source of uncertainty whereas some authors have already dealt with more sources. Huchzermeier and Loch [HuLo01] investigate R&D projects with uncertainty in five areas, namely market payoff, budget, performance, market requirement, and project schedule. Another study by Herath and Park [HePa02] explores R&D investments as compound options with several uncorrelated underlying variables. Brandao et al. [BrDH05] use dynamic programming to solve a binomial decision tree model which allows for including multiple underlying uncertainties. Second, another area for potential development is to consider relationships between follow-on investments. Examples for previous research in the field of project portfolios and ROA are [BaBS04] and [WuOn07]. Third, the probably most promising enhancement would be to account for nested options in addition to ordinary options and compound options (see Figure 35):

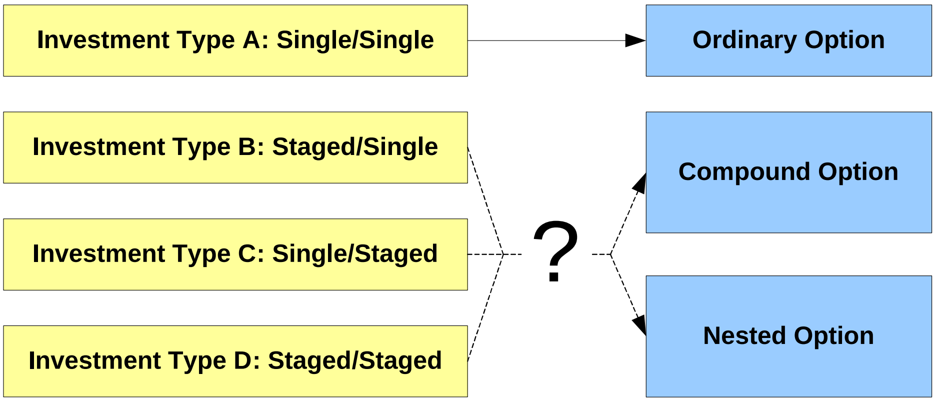

Figure 62: Enhancement of the relationship investment type : option model
Source: Own representation

Compound options are limited to two stages, do not consider direct benefits of the first stage and assume the same stochastic process for both stages. In contrast, nested options can cope with these shortcomings but in general they have no closed form solution. Moreover, the complexity increases when considering interactions between the options. A very good study in the field of nested options is provided by Benaroch et al. [BeSJ06] who extend previous work of Herath and Park [HePa02].

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